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An elasticity-equilibrium-based zigzag theory for axisymmetric bending and stress analysis of the functionally graded circular sandwich plates, using a Maclaurin-type series solution

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The available semi-analytical solutions for bending and stress analysis of the composite/sandwich plates have mainly been proposed for rectangular plates with specific material properties and edge conditions. In the present paper, axisymmetric bending and stress analysis of circular functionally graded sandwich plates subjected to transversely distributed loads is performed. The governing equations are derived based on an elasticity-equilibrium-based (rather than the traditional constitutive-equations-based) zigzag theory. Therefore, both ideas of using the local variations of the displacement field and satisfying a priori the continuity conditions of the transverse stresses at the layer interfaces for predicting the global and local responses of the sandwich circular plates are employed, for the first time. The resulting governing equations are then solved by a semi-analytical Maclaurin-type power-series solution. Each layer of the plate may be made of functionally graded materials. The transverse shear and normal stresses are determined based on the three-dimensional theory of elasticity. Comparisons made with results of a numerical finite element code (ABAQUS software) reveal that even for thick sandwich plates with soft cores, accuracy of results of the present formulation is comparable with that of the three-dimensional theory of elasticity.

1. Introduction

The sandwich plates that are constructed by moving the load carrying high strength face sheets away from the neutral plane or the torsion axis by means of low strength/low density spacer materials (cores) to increase the moments of inertia of the cross section considerably are a type of structures made of discreetly-graded materials. They have been extensively used in the engineering applications. Due to the special situation exists at the interfaces between the face sheets and the core; the available global theories cannot be used to accurately predict the stress distributions and subsequently, the possible failure modes or failure onsets. Furthermore, in some applications the upper layer of the face sheet or core has to be stiffer than the bottom layer; a phenomenon that necessitates employing functionally graded materials (FGMs) for the face sheets or cores.

Some researchers have investigated bending of the functionally graded plates employing the plate theories. Axisymmetric bending and stretching of the functionally graded solid and annular circular plates was studied by Reddy et al. (Reddy et al., 1999) using the first-order shear-deformation Mindlin plate theory. Ma and Wang (Ma and Wang, 2004) employed the third-order shear-deformation plate theory to study axisymmetric bending of the functionally graded circular plates. Saidi et al. (Saidi et al., 2009) and Sahraee and Saidi (Sahraee and Saidi, 2009) studied axisymmetric bending and stretching of functionally graded (FG) circular plates subjected to uniform transverse loadings based on the higher-order shear-deformation plate theories. An analytical solution based on the first-order shear-deformation plate theory was presented by Jomehzadeh et al. (Jomehzadeh et al., 2009) for bending analysis of the functionally graded annular sector plates. Golmakani and Kadkhodayan (Golmakani and Kadkhodayan, 2011) studied axisymmetric non-linear bending of an annular functionally graded plate using the dynamic relaxation method combined with the finite difference technique.

It is evident that results of the three-dimensional theory of elasticity are exact and more accurate than the plate theories which are two-dimensional theories whose dependency on the transverse

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coordinate is prescribed. Assuming the material properties to vary according to exponential laws in transverse and radial directions, Nie and Zhong (Nie and Zhong, 2007) investigated axisymmetric bending of the two-directional functionally graded circular and annular plates based on the three-dimensional theory of elasticity. Li et al. (Li et al., 2008) obtained an elastic solution for axisymmetric bending of FGM circular and annular plates subject to polynomial loads of even order. Yang et al. (Yang et al., 2008) presented an analytical solution for bending of annular plates under uniform loadings. Lei and Zheng (Lei and Zheng, 2009) presented an exact solution for axisymmetric bending of functionally graded circular plates under elastically supported and rigid slipping edge conditions. Based on the three-dimensional theory, Wang et al. (Wang et al., 2010) investigated axisymmetric bending of functionally graded circular plates subjected to Bessel function-type transverse loads. Nie and Zhong (Nie and Zhong, 2010) investigated dynamic behavior of the two-directional FGM annular plates based on the three-dimensional theory of elasticity using the state-space method combined with the differential-quadrature method (DQM). Sburlati and Bardella (Sburlati and Bardella, 2011) obtained three-dimensional elastic solutions for a functionally graded thick circular plate subjected to axisymmetric conditions.

Very limited works have been presented so far for bending analysis of the circular sandwich plates. Liu and Zhu (Liu and Zhu, 1989) proposed a non-linear solution for isotropic circular sandwich plates under uniformly distributed loads. Rabinovitch and Frostig (Rabinovitch and Frostig, 2002) investigated bending behavior of a circular sandwich plate with fully or partially bonded composite laminated face sheets and a soft core. The core was assumed to be a 3D elastic medium and the composite face sheets were modeled using the classical plate theory. Yarovaya (Yarovaya, 2010) studied Bending of metal-polymeric circular sandwich plates resting on Winkler elastic foundations, employing the broken normal hypotheses in the thickness direction. Hou et al. (Hou et al., 2010) analyzed non-linear bending of an isotropic circular sandwich plate using Reissner theory and the point collocation method. The sandwich plate was assumed to be subjected to polynomial distributed loads, uniformly distributed moments, radial pressure or/and radial pre-stress along the edge. Based on ideas of a previous work (Rabinovitch and Frostig, 2002), Santiuste et al. (Santiuste et al., 2011) presented thermo-mechanical non-linear analyses for an axisymmetric semi-isotropic circular sandwich plate with a compliant foam core.

The above brief literature survey reveals that almost only the global (equivalent single layer) theories have been used for bending and stress analysis of the circular plates. Although the traditional zigzag theories have been used in analyzing the rectangular plates to significantly enhance the computation accuracy generally through enforcing the interlaminar transverse stress continuity conditions, they have not been employed for the circular plates. Recently, Shariyat (Shariyat, 2010a, 2010b, 2011a, 2011b, 2011c, 2011d, 2011e, 2012) proposed global-local theories to extend ideas of the zigzag theories for bending, vibration, buckling, and postbuckling of composite, FGM, and viscoelastic plates/shells under thermo-mechanical loads.

In the present paper, axisymmetric bending and stress analysis of circular functionally graded sandwich plates subjected to transversely distributed loads is accomplished. The governing equations are derived based on a zigzag theory whose results are enhanced by employing the three-dimensional theory of elasticity. Therefore, ideas of using the local variations of the displacement field and satisfying the continuity conditions of the transverse stresses at the layer interfaces are employed for the first time for predicting the global and local responses of the circular FGM sandwich plates. The resulting governing equations are then solved by a semi-analytical power-series solution. Each layer of the plate may be made of functionally graded materials. The presented solution covers several boundary conditions and a priori satisfies continuity conditions of the transverse stress components.

2. Development of the basic equations of the zigzag sandwich plate theory

Let us consider the three-layered sandwich plate shown in Fig. 1. Generally, each layer of the sandwich plate may be an FGM layer. Indeed, the displacement components of each point of a specific layer of the plate have to be consistent with the global as well as local stiffness properties of the plate. Therefore, it may be assumed that the local variations of the displacement field are superimposed on the global variations. Subsequently, using linear global and local displacement fields and imposing the kinematic continuity conditions at the interfaces between layers, the displacement field of the entire sandwich plate may be described as follows:

$$\begin{cases} u_1 = u_0 + z\psi_r + \left(z - \frac{h_2}{2}\right)\psi_r^{(1)} + \frac{h_2}{2}\psi_r^{(2)}, & \frac{h_2}{2} \le z \le \frac{h_2}{2} + h_1 \\ u_2 = u_0 + z\psi_r + z\psi_r^{(2)}, & -\frac{h_2}{2} \le z \le \frac{h_2}{2} \\ u_3 = u_0 + z\psi_r + \left(z + \frac{h_2}{2}\right)\psi_r^{(3)} - \frac{h_2}{2}\psi_r^{(2)}, & -h_3 - \frac{h_2}{2} \le z \le -\frac{h_2}{2} \\ w = w_0, \quad -h_3 - \frac{h_2}{2} \le z \le \frac{h_2}{2} + h_1 \end{cases} \tag{1}$$

where u_0 and w_0 are the radial and transverse displacement components of the reference layer (e.g. the mid-surface) of the circular plate, respectively and the coordinate z is measured from the reference layer and is positive upward (Fig. 1). ψ_r is rotation of the normal to the reference surface. Sine it is intended to present a formulation that can show effects of the bending-extension coupling, u_0 and w_0 are not set equal to zero.

For small deflections, the strain–displacement relations may be written as:

$$\varepsilon_r = u_{,r} \quad \varepsilon_\theta = \frac{u}{r} \quad \varepsilon_{rz} = u_{,z} + w_{,r} \tag{2}$$

where the symbol "," stands for the partial derivative. On the other hand, if order of the transverse normal strain is ignorable in comparison to order of the in-plane strains, Hooke's generalized stress-strain law may be expressed as:

$$\sigma_r = \frac{E}{1-\nu^2}(\varepsilon_r + \nu\varepsilon_\theta), \quad \sigma_\theta = \frac{E}{1-\nu^2}(\varepsilon_\theta + \nu\varepsilon_r), \quad \sigma_{rz} = \kappa^2\frac{E}{2(1+\nu)}\varepsilon_{rz} \tag{3}$$

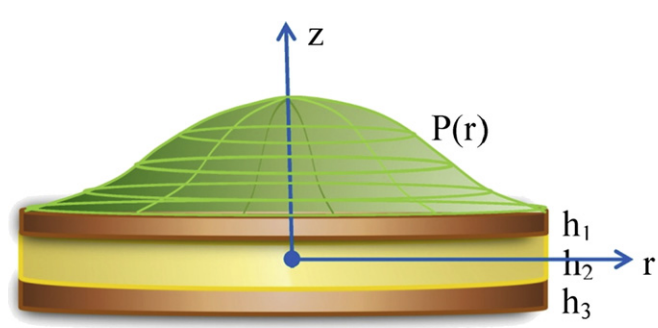

Fig. 1. Kinematic and load parameters of the considered functionally graded circular sandwich plate (boundary conditions are not shown).