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Refined shell model for the linear analysis of isotropic and composite elastic structures

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ABSTRACT

A refined finite element shell model has been developed in this work using an eight-nodes element with nine degrees of freedom for each node. This model enhances the classical shell approaches by including the transverse normal strain. The three displacement components are quadratically expanded in the thickness direction, therefore the transverse shear and normal strains effects are included in such a model making it suitable for thin and thick multilayered composite structures. The transverse normal strain is linear in the thickness direction z and the related shell theory is free from Poisson locking. Finite element locking mechanisms (shear and membrane locking) have been opportunely corrected: good convergence rate has been shown for the considered shell problems (with various geometries, thickness ratios and stacking layer sequences). No shear correction factors are requested.

1. Introduction

In Ramm 1986 shell structures are defined as the primadonna of structures because their behavior is difficult to analyze and they show sensitivity to geometry, support and loading conditions: numerous shell finite elements have been proposed and yet there is a consensus that there are still difficulties in analyzing general shell structures (see the review article by Bischoff et al. 2004). Also, it is difficult to identify which shell elements are the most effective elements currently available, and how to arrive at more efficient shell analysis procedures. In the considerations made in Chapelle and Bathe 1998, it is clear as the presently available finite element schemes for shell analysis are quite well for certain shell problems but they could not properly work for other problems. A shell model is an effective finite element scheme when it is applicable to both categories of shell behavior (namely, the membrane-dominated and bending-dominated cases) and the rate of convergence in either case should be optimal and independent of the shell thickness: typical shear and membrane locking problems. Although the first plate bending element has been introduced in 1961 (MacNeal, 1998), elements which are adequate for general shell analysis only became available in the 1970s. In MacNeal 1998, the author has outlined the chronology of plate and shell elements development remarking the importance of research on the design of plate and shell elements continues to this day. Bischoff and Ramm 2000 have outlined the importance of the development of higher order plate and shell models. These models are able to approximately represent three-dimensional effects, while pertaining the efficiency of a two-dimensional formulation. Especially, they give the possibility to use unmodified, complete three-dimensional material laws within shell analysis, and this is a further major motivation for the development of such models. Babuska and Li 1991 have defined the main requirements to obtain hierarchic models for plates. One of the first 7-parameter shell model has been given by Büchter and Ramm 1992 and it considers the thickness stretch of the shell by including a linear varying thickness stretch as extra variable. In fact, in conventional shell formulations, such as 3- or 5-parameters or even 6-parameters theories, a condensation of the constitutive law is required in order to avoid a significant error due to the assumption of a linear transverse displacement field across the thickness (Büchter et al., 1994). This means that the normal stress in the thickness direction has to either vanish or be constant. In general, these extra constraints cannot be satisfied explicitly or they lead to elaborate strain expressions. This phenomenon (it is not a numerical phenomenon as shear and membrane locking) has been defined as Poisson thickness locking in the 90's, and it has recently been investigated in Carrera and Brischetto (2008a,b) for plate and shell geometries, respectively: when the kinematic model is incoherent

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with the physical constitutive relations, Poisson locking appears. In order to avoid Poisson locking the transverse displacement must be at least quadratically varying in the thickness direction, this means transverse normal strain at least linear in the thickness direction and three-dimensional constitutive equations can be applied. In the case of classical plate/shell theories, the transverse displacement is constant or linear in the thickness direction (zero or constant transverse normal strain) and the reduced constitutive equations must be applied (rearranging them by imposing zero transverse normal stress) (Carrera and Brischetto, 2008a,b). A possibility to remove the Poisson locking effect without increasing the number of degrees of freedom on the structural level has been proposed by Büchter and Ramm 1992, where the strain field of the element is enriched by an additional linear component ε_{33} of the thickness strain. This extra strain is introduced in the sense of a hybrid-mixed formulation by the Enhanced Assumed Strain (EAS) method. Bischoff and Ramm 1997 have proposed a 7-parameter model which considers a linear varying thickness stretch as extra variable, and then an equivalent possibility where a linear expansion in the thickness direction for the in-plane displacement components and a quadratic expansion in the thickness direction for the transverse displacement have been employed. Both these formulation are free of Poisson locking.

By referring to Carrera's Unified Formulation (CUF) (Carrera, 2003), the shell model proposed in the present paper has a quadratic thickness expansion for each displacement component allowing 9 parameters or degrees of freedom for each node. In the framework of CUF several refined plate/shell theories are possible, the 9 parameter theory chosen for this application is enough to avoid the Poisson locking phenomenon (it permits to use three-dimensional constitutive equations) and to correctly describe the behavior of orthotropic and/or moderately thick structures. The proposed model is a refinement and an improvement of the classical degenerative approach presented by Huang and Hinton 1986. Efficiency, generality and simplicity of the use of degenerative approaches have ensured the continued popularity of degenerated shell elements. The basic assumptions employed in classical degenerated shell elements are: – a general curved shell element which has nodes only at the mid-surface; – the stress in the thickness direction is assumed to be equal to zero; – the element displacement field is expressed in terms of three displacements of the mid-surface and two rotations of the mid-surface normal, and in terms of appropriate shape functions. The original degenerated shell element performs reasonably well for moderately thick shell situations. However, for thin shells when full integration is used to evaluate the stiffness matrix, overstiff solutions are often produced due to shear and membrane locking. Shear and membrane locking are the two most important numerical phenomena for shell elements: if a refinement of the mesh is time consuming to contrast them, alternative methods must be proposed in order to correct the overstiff phenomena. In the proposed model the zero transverse normal strain assumption is removed and the basic idea of the degenerative approach is used but with the inclusion of the transverse normal strain. A critical survey of the degenerated shell elements has been proposed by Parisch 1979, the elements are very accurate for both thick and thin plates and shells, and the shear and membrane locking can be contrasted by a reduced integration technique applied only to the membrane and shear stiffness and not to the whole stiffness matrix. The efficiency of the degenerative approach proposed in Huang and Hinton 1986 is also confirmed by its extension to plates and shells with variable thickness values and several boundary conditions (Afonso and Hinton, 1995).

The degenerated approach proposed in this paper considers nine degrees of freedom in each node and the general curved shell element has nodes only in the mid-surface even if the transverse normal strain is linear in the thickness direction. The model is called 9P-8N because an 8-nodes element is considered with 9 degrees of freedom per node (quadratic thickness expansion for each displacement component). Shear and membrane locking corrections are applied as proposed in Polit 1992 and Polit 2008: the stiffness matrix (that of classical isoparametric approach) is partially modified in the parts concerning the transverse shear strains and the in-plane strains which do not tend toward zero in the case of small thickness. Therefore both shear and membrane locking appears as an excessive stiffness value. In the open literature other methods are proposed in order to correct the shear and membrane locking, one of the most powerful is the mixed interpolation of tensorial components approach (MITC) as proposed in Chapelle et al. 2003 and Lee and Bathe 2005. For the same aim Kulikov and Plotnikova 2002 have used the Hu-Washizu mixed variational principle. Shear locking and curvature thickness locking have been treated using the assumed natural strain (ANS) method in Vu-Quoc and Tan 2003, where element is based on the mixed Fraeijs de Veubeke-Hu-Washizu (FHW) variational principle leading to a novel enhancing strain tensor (EAS method) that renders the computation particularly efficient, with improved in-plane and out-of-plane bending behavior (Poisson thickness locking). In Vu-Quoc and Tan 2003, an optimal combination of the ANS method and the minimal number of EAS parameters has been given.

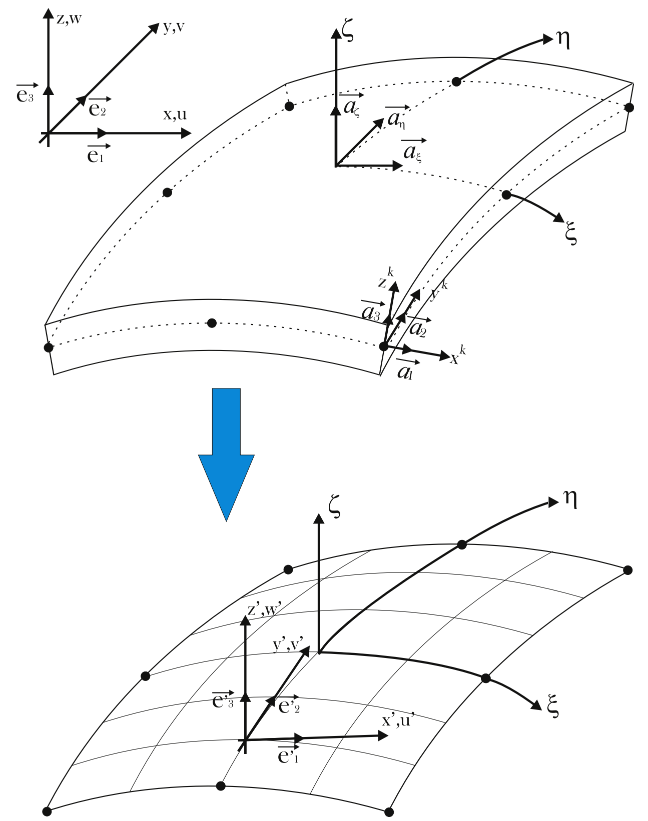

Fig. 1. Employed reference systems in the nine components kinematic model for the eight-nodes shell element (9P-8N).

The main idea of the 9P-8N model is: shell models based on Kirchhoff hypothesis (Kirchhoff, 1850) are suitable when the structure is thin and homogeneous because these conditions permits to respect the hypothesis of zero transverse normal strain and zero transverse shear strains. A refinement of Kirchhoff models is given by those theories based on Reissner–Mindlin (Reissner,