European Journal of Mechanics A/Solids 34 (2012) 130–148

Contents lists available at SciVerse ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

Analytical and numerical studies on Penalized Micro-Dilatation (PMD) theory: Macro-micro link concept

Hamidréza Ramézani [a,*], Holger Steeb [b,**], Jena Jeong [c,1]

[a] CRMD, UMR CNRS 6619 – Research Center on Divided Material, École Polytechnique de l'Université d'Orléans, 8 rue Léonrad de Vinci, 45072 Orléans Cedex 2, France
[b] Institute of Mechanics, Ruhr-University Bochum, Universitätsstrasse 150, D-44 780 Bochum, Germany
[c] ESTP/IRC/LM-Lean Modeling-École Spéciale des Travaux Publics, du Bâtiment et de l'industrie (ESTP), 28 avenue du Président Wilson, 94234 Cachan Cedex, France

ARTICLE INFO

Article history:
Received 21 November 2010
Accepted 7 November 2011
Available online 22 November 2011

Keywords:
Micro-dilatation theory
3D-FEM
Arbitrary Lagrangian–Eulerian method
Porosity change

ABSTRACT

In the present study, the micro-dilatation theory has been investigated. The analytical investigations on the penalized cases using a novel total energy density concept have been achieved via quadratic, affine and linear vector-to-vector and vector-to-scalar mappings. The total energy density has divided into four counterparts which are explicitly linked into the constitutive parameters. This novel energy density concept leads to four explicit constitutive laws. By taking advantage of the strongly ellipticity for the total strain energy density, a freshly defined coupling number was introduced. This issue yields two interesting outcomes, the first one is that the micro-dilatation coupling modulus β is bounded $\beta^2 < K\xi$ and negative ($\beta < 0$) and the second one is that the pore pressure must be less than a specific threshold ($P^2 < \varphi^2 K\xi$). As a matter of fact, the micro-dilatation theory holds a rigorous restriction on the pore-fluid pressure comparing to Biot's theory in which there is no restriction. We have analytically scrutinized the case limits and particularly bulk (case1) and conformal case (case2) in which the heterogeneous deformations occur. The numerical simulations of the micro-dilatation theory have been brought afterwards using Galerkin FEM under an Augmented Lagrangian–Eulerian (ALE) setting. The aforementioned numerical simulations substantiate that one can obtain the physically acceptable results for moderate and low coupling numbers. Based upon these results about the micro-dilatation theory some new routes in determining the micro-dilatation theory constants have been triggered.

1. Overall motivation

1.1. Introduction: historical overview of the micro-mechanical modeling development

In recent years, considerable interest was developed in modeling of micro-structured materials. The micro-mechanical modeling area, initiated more than a century ago by Volterra (Volterra, 1907), began with studies of elastic stress and displacement fields around dislocations and other imperfections. This interest has been motivated by the realization that many materials have heterogeneous micro-structures that play a dominant role in determining macro-deformational behavior. The materials where this occurs include multiphase fiber and particulate composites, soil, rock, concrete, and various granular materials. More recently, using the continuum mechanics principles, theories have been developed in which the material response depends on particular micro-scale length parameters connected with the **existence of inner degrees of freedom** and **nonlocal continuum** behavior[2]. The response of such heterogeneous solids shows strong dependence on the micro-mechanical behaviors among different material phases. The classical theories of continuum mechanics have limited ability to predict such behaviors, and this yields the development of many new micro-mechanical theories of solids. There is a bunch of generalized continuum mechanics which are proposed to handle this goal, e.g. **micro-dilation** by Cowin (Nunziato and Cowin, 1979;

* Corresponding author. Tel.: +33 2 38 257879; fax: +33 2 38 255376.
** Corresponding author. Tel.: +49 234 32 23080; fax: +49 234 32 14229.
E-mail addresses: hamidreza.ramezzani@gmail.com, hamidreza.ramezani@univ-orleans.fr (H. Ramézani), holger.steeb@rub.de (H. Steeb), jeong@profs.estp.fr, jena.jeong@gmail.com (J. Jeong).
[1] Tel.: +33 1 49 082303; fax: +33 1 45 476039.

[2] As a matter of fact, this point is small enough to be considered as a point (infinitesimal) in the classical continuum mechanics and it is equally large enough to provide the micro-effect (to be representative of the micro-structure in a statistical sense) (Neff et al., 2009a).

0997-7538/$ – see front matter © 2011 Elsevier Masson SAS. All rights reserved.
doi:10.1016/j.euromechsol.2011.11.002

Cowin and Nunziato, 1983), **micro-polar** (Günther, 1958; Mindlin, 1964; Eringen et al., 1976; Eringen, 2001) by Günther and Mindlin–Eringen inspired via the Cosserat's brothers landmark work (Cosserat and Cosserat, 1909a, 1909b) in the early of the last century, **micro-stretch** (Eringen, 2001) by Eringen, **micro-strain** (Forest and Sievert, 2006) by Forest and Sievert and **micro-morphic** (Eringen, 2001) by Eringen again. These approaches allow the material deformation to include additional independent micro-structural degrees of freedom. Some fairly recent applications of these theories could be addressed herein, Lakes' landmark studies on the foam and bone (Lakes, 1986, 1995), Diebels and Steeb (Diebels and Steeb, 2002; Diebels and Steeb, 2003) for foams using couple stress theory, Tekoglu and Onck (Tekoglu and Onck, 2008) for Voronoi foams and cellular solids (Tekoglu, 2007), Alsaleh et al. (Alsaleh, 2004; Alsaleh et al., 2006; Alshibli et al., 2006), Khoei and Karimi (Khoei and Karimi, 2008; Karimi and Khoei, 2010), Riahi et al. (Riahi et al., 2009; Riahi and Curran, 2009) for geo-mechanics and Jeong and Ramézani (Jeong et al., 2008; Jeong and Ramézani, 2010) for granular materials.

Along similar lines, Cowin and Nunziato developed a theory of elastic materials with voids or so called micro-dilatation theory including an independent volume fraction change in the constitutive relations (Nunziato and Cowin, 1979; Cowin and Nunziato, 1983; Cowin, 1984a, 1984b, 1985). The Nunziato–Cowin theory has been used in many works to investigate the behavior of deformable porous bodies, e.g., Puri and Cowin (Puri and Cowin, 1985), Ciarletta and Iesçan (Ciarletta and Iesan, 1993), Scarpetta (Scarpetta, 2002), Day et al. (Dey et al., 2004), Bîrsan (Birsan, 2003; Birsan, 2005, 2006a, 2006b) and lately Singh (Singh, 2011).

1.2. Scientific goals: investigation on the micro-dilatation theory for micro-macro link concept

In the current work, we reconsider the linear micro-dilatation theory or void elasticity theory in the quasi-static regime which was originally proposed by Nunziato–Cowin (Nunziato and Cowin, 1979; Cowin and Nunziato, 1983) and Cowin himself (Cowin, 1984a, 1984b, 1985) later on. Let us start presenting the original notation which was proposed by Cowin. Cowin's main objective was to introduce an independent state field variable into the classical theory. The further state field variable well-known as micro-dilatation φ, which was explored before by Goodman and Cowin in (Goodman and Cowin, 1972), describes the volumetric matrix fraction change from the reference state to current state (Fig. 1). The interpretation of the micro-dilatation state field variable in the continuum media with micro-structure viewpoint was given by Capriz and Podio-Guidugli (Capriz and Podio-Guidugli, 1981) and Capriz (Capriz, 1989):

$$\varphi := \Lambda - \Lambda_R \quad \text{where} \quad \Lambda := \frac{\Omega_{\text{matrix}}}{\Omega_{\text{total}}} \tag{1}$$

where, $\Lambda \in]0,1] \subset \mathbb{R}^+$, $\Lambda_R \in]0,1] \subset \mathbb{R}^+$, $\varphi \in]-1,1[\in \mathbb{R}$, $\Omega_{\text{matrix}} \subset \mathbb{R}^3$, $\Omega_{\text{total}} \subset \mathbb{R}^3$ are the volumetric matrix fraction, volumetric matrix fraction at a reference configuration, micro-dilatation, matrix volume and total volume of a porous media including the void inclusions, respectively. Consequently, some new parameters and constitutive laws and equilibrium equations have been appeared.

In the present study, we aim at shedding light on the **micro-dilatation material moduli determination** by means of **penalized cases**. This could be occurred by the comparison between the micro-dilatation theory and Terzaghi's effective stress principle (Terzaghi, 1936) and the bi-phasic poroelastic model consisting of a porous skeleton and an inherent pore-fluid. This matter concludes very interesting outcomes in determining material constants in a rational manner.

The energy density for the micro-dilatation will be conceptually revised in Section 2 and **rational explications** pertaining to the constitutive parameters such as h and g will be developed using the virtual work framework in Section 3. The equilibrium equations will be neatly extracted by the virtual work under weak formulation, which permits us taking into account the Arbitrary Lagrangian–Eulerian (ALE) method for our 3D-FEM simulations in Section 5. The parallel 3D-FEM implementations involving the ALE method will be performed in the paper for **the first time** since Cowin's proposal about the micro-dilatation theory. This would be done due to the fact that we would like to be as close as possible to the realistic simulation via our cutting-edge numerical experiments. We restrict our implementations to infinitesimal deformations due to three reasons: all **penalized cases** which will be discussed herein, can be extracted from **linear kinematics**, second, other groups have not necessarily access to the **geometrically exact or non-linear 3D-FEM** computations and third, most of the practical engineering applications solely require infinitesimal deformations and the extraction of the analytical solution for the non-linear case is extremely difficult because of the **non-linear elliptic operator existence** in the mathematical formulation. The outcomes coming from the experimental determination of the material constants for the micro-dilatation would prepare **a fresh route** in substituting the Biot's theory with the micro-dilatation theory in which the void effects can be captured even for dry porous specimens. Thus, the micro-dilatation

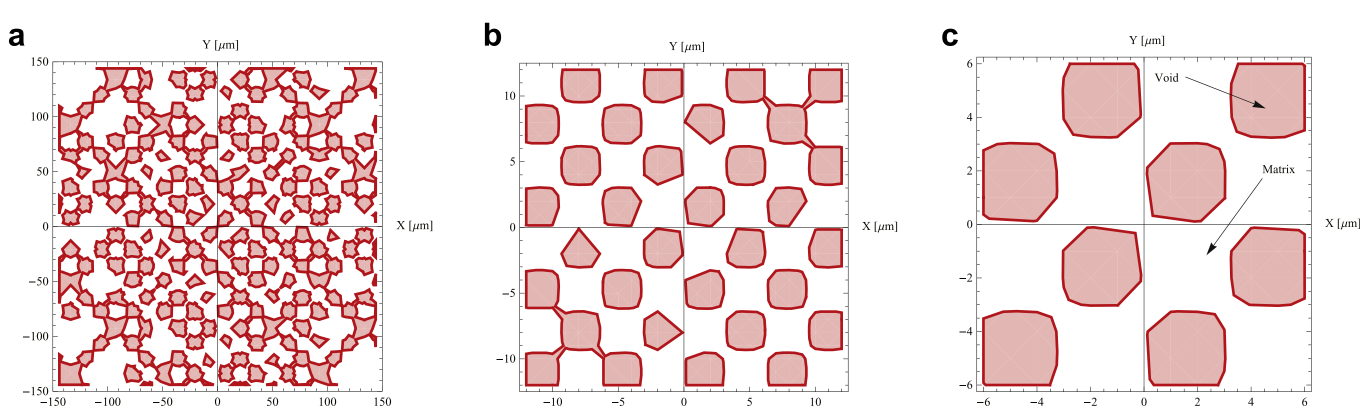

Fig. 1. Typical illustration of void and matrix definition in the micro-dilatation theory or so-called void elasticity, void (pink-colored regions) and matrix (white-colored regions) at a) large scale, b) intermediate scale and c) very small scale. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)