International Journal of Fatigue 89 (2016) 20–35

Contents lists available at ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

Multi-parameter approximation of the stress field in a cracked body in the more distant surroundings of the crack tip

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ARTICLE INFO

Article history:
Received 12 October 2015
Received in revised form 5 February 2016
Accepted 8 February 2016
Available online 17 February 2016

Keywords:
Crack-tip fields
Williams power series
Higher order terms
Stress field reconstruction
Multi-parameter approximation accuracy

ABSTRACT

The accuracy of the multi-parameter approximation of the stress fields in a cracked body is studied in the paper. This approximation, which uses the Williams power series expansion (WE), is intended to be used to estimate the extent of the nonlinear zone, which plays a significant role within tensile failure and the fatigue assessment of non-brittle materials. The characteristics of this zone could be potentially incorporated into methods of determining the true values of fracture parameters and the fatigue behaviour descriptors of materials exhibiting nonlinear failure. Considering the fact that in the case of elastic–plastic and especially quasi-brittle materials the size of this zone is substantial in comparison to specimen dimensions, it is necessary to consider a large region around the crack tip for this task. An automatic routine implemented as a Java application was created to determine the values of coefficients of the higher order terms of the WE that describe crack-tip fields. These values are calculated using the Over-Deterministic Method (ODM), which is applied to the results of the finite element (FE) analysis of an arbitrary mode I test geometry. Furthermore, another Java application developed by the authors provides an analytical reconstruction of the crack-tip stress field by means of the truncated WE, and enables detailed analysis of the crack-tip stress field approximation. The developed procedures simplify the analysis of the description of mechanical fields at a greater distance from the crack tip considerably. The presented research is focused on the optimisation of the selection of FE nodal results entering the ODM procedure used to determine the values of coefficients of the higher order terms of the WE. The aim is to improve the accuracy of the approximation.

1. Introduction

The phenomenon of the nonlinear zone around the crack tip has been recognised within the analysis of the fracture and fatigue behaviour of nonlinearly failing materials for many decades. It has been addressed in fundamental works by Barenblatt [1] (regarding brittle fracture), Dugdale [2] (on yielding materials), and Hillerborg et al. [3] (concerned with concrete); see also e.g. these textbook summaries [4–6] or these more recent specialised papers [7–9]. A typical feature of that zone in these materials is its substantial extent. It is customary to divide the whole nonlinear zone into the plastic zone (PZ), where the material yields and/or hardens, and the fracture process zone (FPZ), where the failure mechanisms resulting in a decrease in material integrity take place. The extent of the PZ is usually characterised by the yield stress of the material (the size of the PZ is proportional to $(K_\mathrm{I}/\sigma_\mathrm{y})^2$ [10]; however, hardening parameters can also be considered in more complex models). Similarly, the size of the FPZ is characterised by material softening properties [11,12].

Types of structural materials differ in the size of the whole nonlinear zone that arises around the crack tip, and also in the proportions of its individual subsets (i.e. the PZ and the FPZ). The most widely used crack tip parameter is the stress intensity factor K, utilizable within the range of applicability of linear elastic fracture mechanics (LEFM) [13]. In metallic materials the high stresses in the vicinity of a crack tip mean that a crack tip plastic zone always develops. However, provided that the PZ is not too large (in the case of small-scale yielding) it is possible to keep using elastic analysis [14,15]. When yielding becomes extensive, the linear elastic approach becomes invalid, and elastic–plastic analysis is needed (i.e. in the case of the large-scale yielding). Within these

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http://dx.doi.org/10.1016/j.ijfatigue.2016.02.016
0142-1123/

Nomenclature

WE	Williams power series expansion, Williams expansion
ODM	over-deterministic method
FE, FEM	finite elements, finite element method
PZ	plastic zone
FPZ	fracture process zone
LEFM	linear elastic fracture mechanics
WST	wedge splitting test
ODeMApp	Over Deterministic Method Application
ReFraPro	Reconstruction of Fracture Process
K_I	stress intensity factor
σ_y	yield strength
$\{\sigma\}$	stress tensor
$\{u\}$	displacement vector
r, θ and x, y	polar and Cartesian coordinates, respectively
n	index of the term of the series
A_n	coefficient of the term of the series
g_n	dimensionless shape function
α	relative crack length
a	crack length
W	specimen width
B	specimen thickness
μ, κ	shear modulus, Kolosov's constant
E, ν	Young's modulus, Poisson's ratio
${}^{\sigma}f_{ij}(n,\theta)$	known functions of WE of the stress tensor $\{\sigma\}$
${}^{u}f_{i}(n,\theta,\kappa)$	known functions of WE of the displacement vector $\{u\}$
σ_appl	nominal stress
$P_\mathrm{sp}, P_\mathrm{v}$	splitting and vertical component of the load, respectively
N	number of terms of the WE considered for the approximation
k	number of selected nodes around the crack tip
u_x, u_y	x-axis and y-axis components of the displacement vector $\{u\}$
σ_{ij}	stress tensor components
σ_eq	equivalent stress
$\sigma_\mathrm{FEM}(x,y)$	value of stress calculated by FEM
$\sigma_\mathrm{WE}(x,y)$	value of stress approximated by WE
$d\sigma(x,y)$	relative error of the stress approximation based on WE
$c, d_\mathrm{n}, h,$	dimension of the test specimen (see Fig. 1)
e, f, i	dimension of the test specimen (see Fig. 1)
S	span between supports
W_ef	effective specimen width
α_w	angle of the slope of the wedge
μ_s	coefficient of friction in roller bearings
$f(x)$	distribution function of distance between selected FE nodes
σ_{yy}	crack opening stress
σ_1	principal tensile stress
β	angle of the segment of the nodal selection
γ	rotation of the segment of the nodal selection

descriptions, the size of the PZ is mainly analysed and the softening zone is ignored due to its negligible relevance in most cases. This contrasts with the case of quasi-brittle materials, where the FPZ is the most relevant part of the nonlinear zone and the insignificant zone of plasticity is ignored.

Estimates of the size, shape and other relevant properties of the nonlinear zone (or its corresponding sub-regions relevant to the investigated material) evolving at the tip of a crack propagated under a static/fatigue regime have not been commonly utilised within the fracture/fatigue analysis of these materials. However, the situation has changed recently. In the case of ductile materials, the dependence of specimen geometry on the PZ has been reported e.g. in [16], where specimens and structures loaded under the same K_I factor exhibited different fracture behaviour and PZ shapes and sizes at crack tips. It has been also shown (e.g. in [17–20]) that stress field description based on K_I is inaccurate and that the utilisation of higher order WE terms for the PZ contour is necessary. In the field of fatigue behaviour, process zone estimation is also dealt with, both experimentally and analytically (e.g. [21,22]).

For quasi-brittle materials, the issue of FPZ extent estimation is also among the main goals of recent research. The evidence of this can again be found both from the experimental [23–25] and computational [26,27] points of view. The work presented in this paper is closely related to this research. It is motivated by the fact that the fracture properties of softening materials exhibit strong dependence on the size and geometry of the specimen, these being known under the terms 'size effect and 'geometry effect, respectively. These effects are mainly linked to the mutual proportions of the internal length of their coarse material structure, the extent of their large FPZ which varies during crack propagation, and the structural dimensions/boundaries. The properties of the FPZ are thus expected to be utilised, among other things, within methods for the evaluation of the 'true' fracture parameters of materials, and their subsequent mechanical characterisation.

Due to these effects (which are also expressed together as the 'boundary effect in some works, e.g. [28]), the investigation the role of the FPZ in the fracture of quasi-brittle materials, particularly concrete, has attracted lot of interest from the engineering community involved in the field of concrete fracture mechanics. In order to estimate FPZ extent, a precise description of the stress state in the cracked body is necessary. For that purpose, multi-parameter fracture mechanics is employed since the size of the FPZ in proportion the whole specimen or structure is much larger in the case of quasi-brittle materials than e.g. the PZ in metals [13,15]. Therefore, the stress field at a greater distance from the crack tip must be described precisely.

The main focus of this contribution is on the testing of two own automatic routines developed by the authors and their team. The first of these enables the computation of the values of coefficients of higher order terms of the WE by applying the ODM to the results of finite element method (FEM) analysis. The second provides an analytical approximation of the stress field in a cracked body via a truncated WE taking into account the values of the coefficients of the higher order terms calculated using the first programmed application. From this point of view, the latter program provides a kind of backward reconstruction of the stress field. In addition to this, additional analyses (e.g. of the approximated stress field accuracy, etc.) are enabled by these applications.

The influence of the number of terms considered for the power series reconstruction of stress fields is investigated in the paper, as well as the way in which FEM nodes are selected for the calculation of coefficients of these terms. Verification of the overall analysis results is conducted on an example of a wedge splitting test (WST) specimen, as such specimens have become very frequently used in the testing of silicate-based composites (which can seen from these recent works, e.g. [29–31] period).

The current paper consists of a theoretical part, where fundamentals of multi-parameter fracture mechanics are summarised, after which the authors own programmed applications used for