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T2R1-type parallel manipulators with bifurcated planar-spatial motion

Grigore Gogu*

Clermont Université, IFMA, EA 3867, Laboratoire de Mécanique et Ingénieries, F-63000 Clermont-Ferrand, France

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This paper presents a new family of *T2R1*-type parallel manipulators with bifurcated planar-spatial motion of the moving platform. In each branch, the moving platform has two independent translations (*T*) and one rotation (*R*) driven by three actuators mounted on the fixed base or on a moving link. The rotation axis is perpendicular to the plane of translations or parallel to this plane. In the first case, the moving platform has a planar motion, and in the second case, a spatial motion. The bifurcation between planar and spatial motion occurs in a constraint singularity in which the connectivity between the moving and fixed platforms increases instantaneously with no change in limb connectivity. Overconstrained/isostatic parallel manipulators with decoupled/uncoupled motions and maximally regular solutions with bifurcated planar-spatial motion of the moving platform are presented for the first time.

1. Introduction

Parallel manipulators (PMs) are composed of an end effector (moving platform) connected to the base (fixed platform) by at least two kinematic chains called limbs. The moving platform of a *TaRb*-type parallel robot can complete $0 \le a \le 3$ independent translational motions (*T*) and $0 \le b \le 3$ rotational motions (*R*) with $2 \le a + b \le 6$.

Rigidity, accuracy, high speed, and high load-to-weight ratio are the main merits of PMs. With respect to serial manipulators, disadvantages include a smaller workspace, complex command and a lower dexterity due to a high motion coupling and multiple singularities inside their workspace. Parallel manipulators with decoupled/uncoupled motions and maximally regular can overcome partially these disadvantages.

Five types of PMs are identified in Gogu (2008a): (i) maximally regular PMs, if the Jacobian J is an identity matrix throughout the entire workspace, (ii) fully-isotropic PMs, if the J is a diagonal matrix with identical diagonal elements throughout the entire workspace, (iii) PMs with uncoupled motions if J is a diagonal matrix with different diagonal elements, (iv) PMs with decoupled motions, if J is a triangular matrix and (v) PMs with coupled motions if J is neither a triangular nor a diagonal matrix. This paper focuses on solutions with decoupled/uncoupled motions and maximally regular as well.

T2R1-type PMs are used in applications that require two independent planar translations (*T2*) and one independent rotation (*R1*) of the mobile platform around an axis perpendicular to the plane of translations or parallel to this plane. In the first case, the moving platform has a planar motion, and in the second case, a spatial motion.

Usually, distinct architectures of *T2R1*-type parallel robots are used to obtain planar or spatial motion of the moving platform with a given position of the rotation axis. Various architectures are known in the literature to obtain three-legged planar or spatial parallel manipulators (Gogu, 2008a, 2009a; Hunt, 1982; Kong and Gosselin, 2005, 2007; Merlet, 2006). The major applications of this type of PMs are: flight and motion simulation, pointing and tracking, assembling and machining.

The solutions presented in this paper have the advantage of using the same structural architecture which can bifurcate between planar and spatial motion of the moving platform. This bifurcation occurs in a constraint singularity in which the connectivity between the moving and fixed platforms increases instantaneously with no change in limb connectivity.

The classical methods used for structural synthesis of PMs can be divided into four approaches: (i) the methods based on displacement group theory (Angeles, 2004; Hervé, 1995, 1999; Huynh and Hervé, 2005; Lee and Hervé, 2006; Rico et al., 2006), (ii) the methods based on screw algebra (Kong and Gosselin, 2005, 2007; Tsai, 1999; Frisoli et al., 2000; Huang and Li, 2002), (iii) the method based on velocity loop equations (Di Gregorio and Parenti-Castelli, 1998) and (iv) the method based on the theory of linear transformations (Gogu, 2004a, b, 2006, 2007, 2008a, 2009a, b).

The structural synthesis approach presented in this paper is founded on the theory of linear transformations and integrates the new formulae for mobility, connectivity, redundancy and overconstraint of parallel manipulators proposed in Gogu (2005a, 2008a).

* Tel.: +33 4 73288022; fax: +33 4 73288100.
E-mail address: Grigore.Gogu@ifma.fr.

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The main aims of this paper are to present for the first time a family of *T2R1*-type overconstrained/isostatic parallel manipulators with decoupled/uncoupled motions along with maximally regular solutions with bifurcated planar-spatial motion of the moving platform.

2. Structural parameters of parallel manipulators

The main parameters used for structural synthesis of parallel robots are associated with mobility, connectivity, redundancy and overconstraint of parallel mechanisms (Gogu, 2008a).

Mobility is the main structural parameter of a mechanism and also one of the most fundamental concepts in the kinematic and dynamic modelling of mechanisms (Gogu, 2005b). IFToMM terminology defines the mobility or the degree of freedom as the number of independent coordinates required to define the configuration of a kinematic chain or mechanism (Ionescu, 2003).

The various methods proposed in the literature for mobility calculation of the closed loop mechanisms fall into two basic categories:

(a) approaches for mobility calculation based on setting up the constraint equations and calculating their rank for a given position of the mechanism with specific joint locations,
(b) formulae for a quick calculation of mobility with no need to develop the set of constraint equations.

The approaches used for mobility calculation based on setting up the constraint equations and their rank calculation are valid without exception. The major drawback of these approaches is that the mobility cannot be determined quickly without setting up the kinematic or static model of the mechanism. Usually, the kinematic model is expressed by the closure equations that must be analyzed for dependency. The information about mechanism mobility is usually derived by performing position, velocity or force analysis by using various analytical tools (screw theory, linear algebra, affine geometry, Lie algebra, etc). For this reason, the real and practical value of these approaches is very limited in spite of their valuable theoretical foundations. Moreover, the rank of the constraint equations is calculated in a given position of the mechanism with specific joint locations. The mobility calculated in relation to a given configuration of the mechanism is an instantaneous mobility which can be different from the general mobility (global mobility or full-cycle mobility). Global mobility is a global parameter characterizing the mechanism in all configurations of a free-of-singularity branch. Instantaneous mobility is a local parameter characterizing the mechanism in a given configuration including singular ones. In a singular configuration, the instantaneous mobility could be different from the global mobility.

A formula for quick calculation of mobility is an explicit relationship between the following structural parameters: the number of links and joints, the motion/constraint parameters of joints and of the mechanism. Usually, these structural parameters are easily determined by inspection with no need to develop the set of kinematic or static constraint equations.

The classical formulae for a quick calculation of mobility known as Chebychev–Grübler–Kutzbach formulae do not fit many classical mechanisms and recent parallel robots. These formulae have been recently reviewed in Gogu (2005b) and their limits have been set up in Gogu (2005c). New formulae for quick calculation of the mobility have been proposed in Gogu (2005a) and demonstrated via the theory of linear transformations. A development of these contributions can be found in Gogu (2008a).

The connectivity between two links of a mechanism represents the number of independent finite and/or infinitesimal displacements allowed by the mechanism between the two links.

The number of overconstraints of a mechanism is given by the difference between the maximum number of joint kinematic parameters that can lose their independence in the closed loops, and the number of joint kinematic parameters that actually lose their independence in the closed loops.

The redundancy is given by the difference between the mobility of the parallel mechanism and the connectivity of the end-effector. Redundancy introduces internal mobilities in the limbs which are given by the difference between limb mobility and connectivity.

Let us consider the case of a parallel mechanism $F \leftarrow G_1\text{–}\ldots\text{–}G_j\text{–}\ldots\text{–}G_k$ in which a characteristic link (end-effector) $n \equiv n_{G_j}$ is connected to a reference link $1 \equiv 1_{G_j}$ by k simple or complex kinematic chains $G_j\ (1_{G_j} - 2_{G_j} - \ldots - n_{G_j})$ called limbs ($j = 1, 2, \ldots, k$). The identity symbol "$\equiv$" is used between the links to indicate that they are welded together in the same link. For example, the notation $1 \equiv 1_{G_j}$ indicates that the first link of each limb is the fixed platform. The mechanism $F \leftarrow G_1\text{–}\ldots\text{–}G_j\text{–}\ldots\text{–}G_k$ is characterized by:

R_{G_j} – the vector space of relative velocities between the distal links n_{G_j} and 1_{G_j} in the kinematic chain G_j disconnected from parallel mechanism F,

R_F – the vector space of relative velocities between the distal links $n \equiv n_{G_j}$ and $1 \equiv 1_{G_j}$ in the parallel mechanism $F \leftarrow G_1\text{–}\ldots\text{–}G_j\text{–}\ldots\text{–}G_k$,

$S_{G_j} = dim(R_{G_j})$ – the connectivity between the distal links n_{G_j} and 1_{G_j} in the kinematic chain G_j disconnected from parallel mechanism F,

$S_F = dim(R_F)$ – the connectivity between the distal links $n \equiv n_{G_j}$ and $1 \equiv 1_{G_j}$ in the parallel mechanism $F \leftarrow G_1\text{–}\ldots\text{–}G_j\text{–}\ldots\text{–}G_k$.

These parameters are related by the new formulae for mobility M_F, connectivity S_F, overconstraint N_F, and redundancy T_F of the parallel mechanism $F \leftarrow G_1\text{–}\ldots\text{–}G_j\text{–}\ldots\text{–}G_k$ proposed in Gogu (2008a):

$$M_F = \sum_{i=1}^{p} f_i - r \tag{1}$$

$$N_F = 6q - r \tag{2}$$

$$T_F = M_F - S_F \tag{3}$$

with

$$S_F = dim(R_F) = dim\left(R_{G_1} \cap R_{G_2} \cap \ldots \cap R_{G_k}\right) \tag{4}$$

$$r = \sum_{i=1}^{k} S_{G_i} - S_F + r_l \tag{5}$$

and

$$r_l = \sum_{i=1}^{k} r_l^{G_i} \tag{6}$$

where p represents the total number of joints, $q = p - m + 1$ is the total number of independent closed loops in the sense of graph theory, m is the total number of members of the parallel mechanism including fixed base, f_j is the mobility of the jth joint, r – the number of joint parameters that lose their independence in parallel mechanism F, $r_l^{G_j}$ – the number of joint parameters that lost their independence in the closed loops of limb G_j, r_l – the total number of joint parameters that lost their independence in the closed loops that may exist in the limbs of the parallel mechanism $F \leftarrow G_1\text{–}\ldots\text{–}G_j\text{–}\ldots\text{–}G_k$.