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The Modified Manson–Coffin Curve Method to estimate fatigue lifetime under complex constant and variable amplitude multiaxial fatigue loading

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ABSTRACT

This paper investigates the accuracy of the so-called Modified Manson–Coffin Curve Method (MMCCM) in estimating fatigue lifetime of metallic materials subjected to complex constant and variable amplitude multiaxial load histories. The MMCCM postulates that fatigue damage is maximised on that material plane experiencing the maximum shear strain amplitude. In the present investigation, the orientation of the critical plane was determined through that direction along which the variance of the resolved shear strain reaches it maximum value. Under variable amplitude complex load histories, this direction was also used to count the resolved shear strain cycles via the classic Rain-Flow method. Further, the degree of multiaxiality and non-proportionality of the time-variable stress states at the assumed critical locations was directly quantified through a suitable stress ratio which accounts for (i) the mean value and the variance of the stress perpendicular to the critical plane as well as for (ii) the variance of the shear stress resolved along the direction experiencing the maximum variance of the resolved shear strain. The accuracy and reliability of the proposed approach was checked against approximately 650 experimental data taken from the literature and generated by testing un-notched metallic materials under complex constant and variable amplitude multiaxial load histories. The sound agreement between estimates and experimental results which was obtained strongly supports the idea that the proposed design technique is a powerful engineering tool allowing metallic materials to be designed against constant and variable amplitude multiaxial fatigue by always reaching a remarkable level of accuracy. This approach offers a complete solution to the strain based multiaxial fatigue problem.

1. Introduction

In situations of practical interest, engineering components and structures are subjected to complex time-variable load histories, the applied time-dependent systems of forces/moments resulting in local variable amplitude (VA) multiaxial stress/strain states. Estimating fatigue strength of metallic materials subjected to VA multiaxial load histories is a complex design problem which must be addressed properly in order to avoid unwanted breakages during in-service operations. Owing to the high costs associated with fatigue failures, since the beginning of the last century a tremendous effort has been made by the international scientific community to devise appropriate engineering tools suitable for estimating fatigue damage under complex loading paths. If attention is focused on the low/medium-cycle fatigue regime, examination of the state of the art [1–7] suggests that, so far, this intractable design problem has being addressed mainly by trying to extend the use of well-known constant amplitude (CA) multiaxial fatigue criteria to those situations involving multiaxial VA load histories. In this context, among the methods which have been employed so far, certainly the SWT parameter [8,9], Brown & Miller's criterion [10,11], and Fatemi & Socie's critical plane approach [12,13] deserve to be mentioned explicitly.

As far as VA multiaxial load histories are concerned, accurately performing the cycle counting certainly represents one of the trickiest aspects, the scientific community being still debating to agree a commonly accepted strategy. As to the cycle counting issue, examination of the state of the art suggests that the most successful methodologies [11,13–15] which have been formalised and

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validated so far are all based on the use of the classic Rain-Flow method (this method being originally developed to address simple uniaxial situations [16]).

When it comes to designing components and structures against VA multiaxial fatigue, another tricky problem that must be addressed properly is the definition of an appropriate rule suitable for estimating cumulative damage. Even though a variety of methods have been proposed so far [17], certainly, in situations of practical interest, the most used rule is still the linear one devised by Palmgren [18] and Miner [19]. According to this classic approach, fatigue failure takes place as soon as the damage sum becomes equal to unity. However, accurate experimental investigations have proven that the critical value of the damage sum, D_{cr}, vary in the range 0.02–5, its average value being equal to 0.27 for steel and to 0.37 for aluminium [20]. Further, given the material, D_{cr} is seen to vary as the geometry of the component, the degree of multiaxiality of the assessed VA load history, and the profile of the considered load spectrum change [20–22]. Thus, systematically taking the critical value of the damage sum equal to unity may lead, under particularly unfavourable circumstances, to non-conservative estimates. This suggests that D_{cr} can be evaluated accurately for the specific material/component/load history being assessed solely via expensive and time-consuming experimental trials.

In this complex scenario, this paper reports on an attempt of extending the use of a multiaxial fatigue criterion we have recently proposed [23–25] – here called the Modified Manson–Coffin Curve Method (MMCCM) – to those situations involving complex CA and VA loading paths. In more detail, such a strain based critical plane approach is attempted here to be applied along with the maximum variance concept [26–28] in order to formalise a robust fatigue assessment technique suitable for estimating fatigue lifetime of metallic materials subjected to complex CA and VA multiaxial load histories.

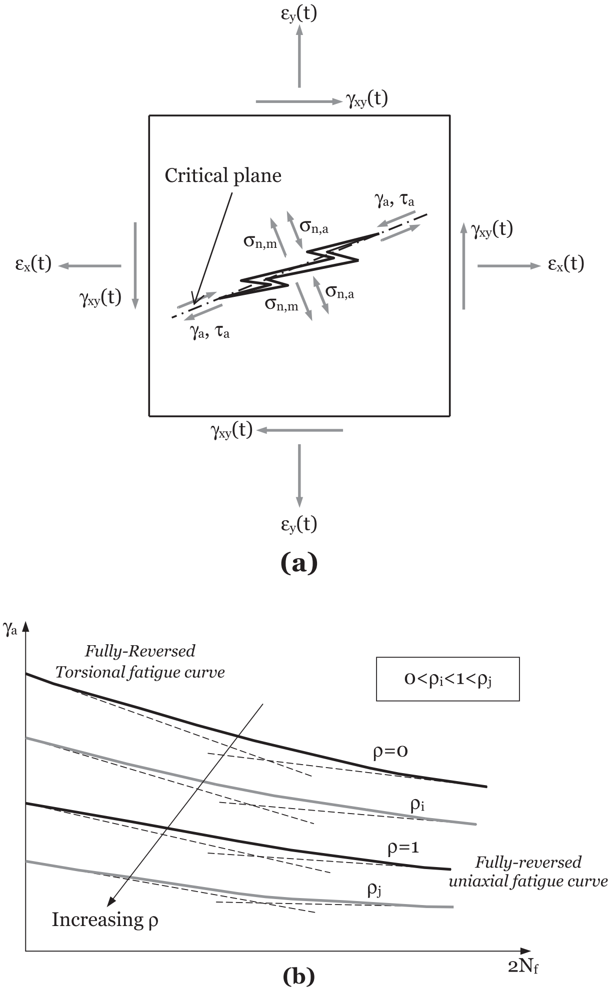

Fig. 1. Fatigue damage model (a) and Modified Manson–Coffin diagram (b).

2. Fundamentals of the MMCCM

As far as CA loading paths are concerned, the MMCCM [23–25] postulates that fatigue damage in the low/medium-cycle fatigue regime can accurately be estimated via the stress and strain components acting on that material plane (i.e., the so-called critical plane) experiencing the maximum shear strain amplitude, γ_a. The degree of multiaxiality and non-proportionality of the applied load history as well as the presence of non-zero mean stresses are quantified by the MMCCM via the shear stress amplitude, τ_a, relative to the plane of maximum shear strain amplitude and the amplitude, $\sigma_{n,a}$, and the mean value, $\sigma_{n,m}$, of the stress normal to the critical plane. The definitions which are proposed here as being adopted to calculate the stress/strain quantities of interest not only under CA, but also under VA multiaxial fatigue loading will be discussed in the next section in great detail.

The fatigue damage model on which the MMCCM is based is shown in Fig. 1a. According to this schematisation, Stage I cracks are assumed to initiate on those crystallographic planes most closely aligned with the maximum shear strain direction [29]. The subsequent propagation phenomenon is strongly influenced by the stress perpendicular to the critical plane [9,30]. In particular, the amplitude of the stress normal to the critical plane, $\sigma_{n,a}$, favours the growth process by cyclically opening and closing the micro/meso fatigue cracks [31]. The propagation phase is also influenced by the mean stress, $\sigma_{n,m}$, normal to the plane of maximum shear strain amplitude. In fact, a tensile superimposed static normal stress tends to keep the micro/meso fatigue cracks open by minimising the interactions amongst the crack surfaces' asperities [9,30]. This favours the effect of the cyclic shear stress which pushes the tips of the cracks themselves [32]. On the contrary, under compressive mean normal stresses, the resulting additional frictional phenomena between the crack surfaces [9,30] mitigate the action of the cyclic shear stress [32], this resulting in a reduction of the crack growth rate.

According to the fatigue damage model depicted in Fig. 1a, the degree of multiaxiality and non-proportionality of the stress state damaging the assumed crack initiation locations is quantified by the MMCCM via the following critical plane stress ratio [23]:

$$\rho = \frac{\sigma_{n,m} + \sigma_{n,a}}{\tau_a} = \frac{\sigma_{n,\max}}{\tau_a} \tag{1}$$

In definition (1) τ_a is the shear stress amplitude relative to the critical plane, whilst $\sigma_{n,m}$, $\sigma_{n,a}$ and $\sigma_{n,\max}$ are the mean value, the amplitude and the maximum value of the stress perpendicular to the plane of maximum shear strain amplitude, respectively. Ratio ρ is seen to be capable of modelling not only the presence of superimposed static stresses, but also the degree of multiaxiality and non-proportionality of the applied load history [23,31]. In particular, as suggested by Socie [9,30], the effect of the stress components perpendicular to the critical plane can efficiently be modelled by simply using the maximum normal stress, since $\sigma_{n,\max} = \sigma_{n,m} + \sigma_{n,a}$. This simple strategy was followed by Socie himself to reformulate the SWT parameter to make it suitable for performing the multiaxial fatigue assessment of those metals whose mesoscopic cracking behaviour is mainly Mode I governed [9]. Similarly, the normal maximum stress, $\sigma_{n,\max}$, was employed