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A correction in the algorithm of fatigue life calculation based on the critical plane approach

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ABSTRACT

The paper presents the algorithm for calculating the fatigue life taking into account the variability of coefficients occurring in the multiaxial fatigue criterion depending on the number of cycles to failure. The algorithm has been analysed under uniaxial cyclic loads and a combination of bending and torsion for four structural materials. Significant increase of convergence of calculated and experimental fatigue life using the new algorithm as compared to the classical approach for five selected multiaxial fatigue criteria based on a critical plane has been demonstrated.

1. Introduction

Fatigue defined as the degradation of mechanical properties of the material under loadings which vary in time is one of the main causes of the limited life time of machines and structures. This results in an increase in operating costs and is the reason for ongoing research on the complex phenomenon of fatigue failure. One research areas is the multiaxial fatigue criteria which aim at evaluating the fatigue degradation of the material at any load run. This evaluation is usually carried out by comparing the reduced complex state of stress to a scalar value equivalent to a uniaxial stress (the so-called fatigue limit). The proposed function for reducing the complex stress to a uniaxial state is an essential part of the multiaxial fatigue criterion. Among the many proposed functions, one can distinguish a group characterized by the assumption that the components of the stress associated with the plane of a certain orientation are responsible for the initiation of fatigue cracks. The orientation of the plane should coincide with the plane of the fatigue crack. This proposal, known as the critical plane concept, has gained great interest in the academia [1–8]. Despite the considerable amount of literature and research on this concept, there is no proposal of a criterion accepted by the wider group of researchers and applying to different materials and loads.

The reduction functions proposed in the criteria are also used to calculate the fatigue life N_{cal} by comparing the equivalent value of stress σ_{eq} to stress $\sigma(N_f)$ from the fatigue characteristics (e.g. Wöhler or Basquin), assuming that $N_{cal} = N_f$. Fatigue characteristics obtained during cyclic torsion $\sigma(N_f) = \tau_f(N_f)$, tension–compression, and bending $\sigma(N_f) = \sigma_f(N_f)$ are the most commonly used. Correctly proposed reducing function applied to any case of a uniaxial load, for example torsion, tension–compression, or bending stresses, but with the same fatigue life, brings these stresses to the equivalent state, thus

$$\sigma_{eq}(N, \text{Torsion}) = \sigma_{eq}(N, \text{Bending}) = \sigma(N = N_f). \quad (1)$$

Reducing functions based on the critical plane are usually linear or non-linear function of material parameters and shear τ_{ns}, normal σ_n (in the critical plane), or hydrostatic σ_h stresses (invariant of the stress). Material parameters are determined in such a way that the stress reduction satisfies Eq. (1). Typically, the fatigue criteria in their original form are proposed to assess the limit state, that is, for the so-called fatigue limit. Simplifying the problem by adopting the theoretical fatigue limit (for steel) σ_{af} corresponding to the number of cycles $N = 2{\cdot}10^6$, that is, $\sigma(N_f = 2{\cdot}10^6) = \sigma_{af}$, Eq. (1) is reduced to

$$\sigma_{eq}(\text{Torsion}) = \sigma_{eq}(\text{Bending}) = \sigma_{af}. \quad (2)$$

Therefore, the material parameters are relations of fatigue limits from uniaxial stress states. Applying the proposed reduction function to the fatigue life other than the limiting one requires looking for material parameters which satisfy Eq. (1). Unfortunately, the fatigue criteria, or rather the reducing functions used to calculate the so-called reduced fatigue life ($N < 2{\cdot}10^6$ for steel)

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Nomenclature

A_σ, m_σ	parameters of a linear regression for fatigue characteristic under cyclic bending
A_τ, m_τ	parameters of a linear regression for fatigue characteristic under cyclic torsion
a, b, k	material constants
E	Young's modulus
Er	residual of aim function
N	number of cycles
$\mathbf{n}$	unit vector normal to the analysed plane orientation
$\mathbf{s}$	unit vector pointing the analysed shear direction perpendicular to $\mathbf{n}$
$R_{eH}/R_{p0.2}$	upper yield strength/proof strength
R_m	tensile strength
ν	Poisson coefficient
σ	stress
τ	shear stress
$Pr(T)$	number of point in space $N_{cal} - N_{exp}$ included inside scatter band T divided by total number of points (empirical probability)
T	Scatter band: $T = \frac{N_{exp}}{N_{cal}}$ for $N_{exp} > N_{cal}$; $T = \frac{N_{cal}}{N_{exp}}$ for $N_{exp} \leqslant N_{cal}$
$T(Pr)$	scatter band for which Pr ratio is included
T_{exp}	scatter band for $Pr = 0.95$ determined only for uniaxial loading
t	time

Subscripts

af	fatigue limit
cal	calculated
c	critical (at failure)
d	index to select: n, a or h, max
eq	equivalent
exp	experimental
f	from fatigue characteristic
h	hydrostatic
m	mean value
max	maximum value
n	normal (in direction of $\mathbf{n}$)
ns	shear (on the plane with normal $\mathbf{n}$ in $\mathbf{s}$ direction)
$p1$, $p2$, $p3$	parameters to select the analysed criterion

are usually assumed with coefficients which are the functions of fatigue limits [9–13]. This approach is valid only for a certain class of materials, for which

$$\frac{\sigma_f(N_f)}{\tau_f(N_f)} = const, \tag{3}$$

that is for materials with parallel fatigue characteristics. This fact was noted in several papers including [14–16].

The aim of this paper is to propose an algorithm for determining the fatigue life by using generally known fatigue criteria, taking into account the correct determination of material parameters which are a function of the number of cycles to failure. Validation of the proposed algorithm is performed using stress based criteria applicable in the high cyclic fatigue regime. However, the main idea of correction could be implemented also in strain or energy based criteria.

2. A brief description of analysed multiaxial fatigue criteria

2.1. Stanfield (1935), Stullen-Cummings (1954), Findley (1959) criterion: C1

Stanfield [17] was the first to propose the calculation of the critical shearing stress value τ_c (fatigue strength, limiting value for failure) for a multiaxial stress state based on a linear combination of the shear τ_{ns} and normal σ_n stresses in the plane of the material at a certain orientation

$$\tau_c = \max_{\mathbf{n}}\{\tau_{ns,a} + k\sigma_{n,a}\}, \tag{4}$$

where k is a material constant. According to Stanfield, τ_c is calculated in a plane (with normal $\mathbf{n}$) on which a linear combination of (4) is at the maximum. Stanfield has not verified this proposal experimentally nor subjected it to a more extensive analysis. The same Eq. (4) has been proposed by Stulen and Cummings [18] by citing the similarities to the static Mohr–Coulomb criterion. Stulen and Cummings have analyzed Eq. (4) in uniaxial and complex stresses by deriving the relationship between the constant k, the fatigue limit, and the value of τ_c under cyclic torsion. They have also done the experimental verification by examining the correlation of τ_c with the number of cycles to failure. The concept included in Eq. (4) has been extended by Findley [19], who has taken the (in time domain) maximum of the normal stress $\sigma_{n,\max}$ into account,

$$\tau_c = \max_{\mathbf{n}}\{\tau_{ns,a} + k\sigma_{n,\max}\}, \tag{5}$$

where $\sigma_{n,\max} = \sigma_{n,a} + \sigma_{n,m}$, $\sigma_{n,a}$ and $\sigma_{n,m}$ are the amplitude and the mean value of normal stress, respectively. Findley et al. have pointed out in [15] that the k constant depends on the number of cycles to failure N and this relationship has the following form for the reduction of stress according to (4)

$$\frac{\sigma_f(N_f)}{\tau_f(N_f)} = \frac{2}{1 + k/\sqrt{1 + k^2}}, \tag{6}$$

where $\sigma_f(N_f)$ and $\tau_f(N_f)$ are fatigue characteristics relevant for bending and torsion. When looking for the maximum value of (4) as a function of orientation of the analysed plane and comparing the resulting value to uniaxial stresses, i.e. torsion and tension (bending), it turns out that τ_c is proportional to τ_{af}. After the appropriate transformations, the fatigue criterion based on (4) can be described by the following expression

$$\max_{\mathbf{n}}\{a\tau_{ns,a} + b\sigma_{n,\max}\} \leqslant \tau_{af}, \tag{7}$$

where t is the time, $\mathbf{n}$ is a vector describing the orientation of the analysed plane, $\mathbf{s}$ is the vector normal to $\mathbf{n}$ describing the direction of the shear stress τ_{ns},

$$a = \frac{2\sqrt{\tau_{af}\sigma_{af} - \tau_{af}^2}}{\sigma_{af}}, \quad b = \frac{2\tau_{af}}{\sigma_{af}} - 1 \quad \text{for} \quad 1 \leqslant \frac{\sigma_{af}}{\tau_{af}} \leqslant 2. \tag{8}$$

2.2. Matake criterion: C2

Matake [20] has simplified the criterion (7) by changing the definition of the critical plane. The critical plane in the Matake concept is the plane of maximum shear stress,

$$\max_{t,\mathbf{n},\mathbf{s}}\{\tau_{ns}\} + k\max_{t}\{\sigma_n\} \leqslant \tau_{af}. \tag{9}$$