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Size effect in Paris law and fatigue lifetimes for quasibrittle materials: Modified theory, experiments and micro-modeling

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A B S T R A C T

Unlike brittle materials, quasibrittle materials exhibit a structure size effect on the fatigue crack growth rate, particularly on the Paris law coefficient (prefactor). This size effect is strong for specimens not much larger than the dominant material inhomogeneities (or aggregate sizes in concrete), and vanishes for very large structures. It can be quantified by a size adjustment of Paris law which is similar to the size effect law for monotonic loading. But the transitional size D_{0c} at which the transition is centered is not the same. Previous experiments aimed at quantitative analysis of this size effect involved only one or a few specimens per size. Thus the huge scatter, inevitable in fatigue tests, distorted the estimates of D_{0c} and, thereby, also of the size of the cyclic fracture process zone (FPZ), to which D_{0c} is proportional. Here, more reliable estimates of D_{0c} and the cyclic FPZ size are obtained by conducting, on concrete, multiple fatigue tests per size and taking the average. Furthermore, these length characteristics are also estimated numerically using the latest version of the microplane constitutive damage model for concrete (model M7), extended to quasibrittle fatigue. It is conclusively shown that the cyclic FPZ is smaller than the monotonic FPZ. Further, the numerical simulations of the cyclic deformations within the FPZ reveal that the D_{0c} obtained from the previous form of the size-adjusted Paris law is not proportional to the cyclic FPZ size. A new form is proposed and verified by updated dimensional analysis. It involves the transitional sizes for both the monotonic and cyclic size effects and is seen to yield values of D_{0c} that are proportional to the cyclic FPZ size. The ensuing size effect on fatigue lifetimes is simulated using both the previous and new forms. Both forms are found to predict a non-monotonic size effect on the lifetimes, initially decreasing and, after a minimum, eventually increasing with increasing size. It is also shown that, similar to Paris law for brittle fatigue, an extension to the fatigue threshold at nearly vanishing amplitude is impossible because of a transition to Charles–Evans law for static fatigue. Finally, a possible ramification to fatigue of micrometer-scale metallic devices is pointed out.

1. Introduction

The mechanics of fracture and fatigue is of three kinds: brittle, brittle–ductile and quasibrittle. In brittle fracture mechanics, conceived by Griffith in 1921, the fracture process zone (FPZ) can be considered to be a point and the specimen or structure follows linear elastic fracture mechanics (LEFM). In brittle–ductile fracture, most of the FPZ is plastic although the FPZ can still be treated almost as point embedded in the plastic zone. In quasibrittle fracture [1, e.g.], most of the nonlinear zone surrounding the crack tip represents the FPZ and undergoes progressive softening damage. The fact that the FPZ size in quasibrittle materials is not negligible compared to structural dimensions gives rise to transitional size effects. The size effect in quasibrittle fatigue was first demonstrated in [11,12]. Recently it was carefully analyzed for rock by Le et al. [13]. Their work is here extended to concrete and the use of microplane model makes it possible to resolve the inelastic phenomena within the FPZ governing the fatigue crack growth.

Fatigue crack growth in a wide variety of brittle materials is described well by the famous Paris (or Paris–Ergodan) law [14], which relates the rate of crack growth per cycle da/dN to the amplitude of the stress intensity factor ΔK via a power law, and is expressed as,

$$\frac{da}{dN} = C(\Delta K)^m \tag{1}$$

where C and m are material parameters dependent on the load ratio R, the environment, etc. [15]. This law was shown to be applicable to quasibrittle materials as well [11–13]. These heterogeneous

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materials with brittle constituents include concrete, as the archetypical case, many rocks, tough ceramics, fiber composites (and even metals at the micrometer scale). Quasibrittle fatigue crack growth is characterized by a large FPZ that is non-negligible compared to structural dimensions, and its localization is limited by a finite material characteristic length. This engenders a structural size effect on the fatigue crack growth, causing a size effect in Paris law and consequently on the fatigue lifetimes. This has important implications for the fatigue design and lifetime safety factors of quasi-brittle structures in a variety of civil, aerospace, energy and even semi-conductor applications.

The topic of scaling in Paris law has been studied for metals by many researchers; e.g., [2–7] but mainly from an experimental standpoint. Some researchers have also analyzed this problem using dimensional analysis [8–10]. But very few studies aimed at a truly quantitative understanding of this size effect; see [11–13]. These led to the development of a 'size adjusted' Paris law.

It was also inferred in these studies that the size of the cyclic FPZ is different from the monotonic one. However there was no general consensus about how different it was. The reason was that the aforementioned studies failed to take into account the experimental scatter, which is huge for fatigue. Further, they did not study the size effect in lifetime. Thus, the goals of this study are:

1. **Experimental verification** of the existing size adjusted Paris law, specifically of the estimates of the transitional size D_{0c} and the cyclic FPZ size. The fatigue experiments considered were performed on geometrically scaled concrete beams of multiple sizes, with multiple specimens per size. The average response of several specimens will be analyzed so as to minimize the influence of scatter.
2. **Numerical verification and FPZ resolution**, based on the recently developed damage model for quasibrittle fatigue [16], which is a refinement of microplane model M7. The numerical approach is expected to predict the mean behavior with reduced scatter. Furthermore, it makes possible accurate determination of the physical FPZ sizes, which provides additional means for verification.
3. **Size effect extension** to fatigue lifetimes, resulting from the size effect on Paris law.

2. Existing form of the size adjusted Paris law

The size effect on structural strength of brittle and quasibrittle materials is by now well understood [1,17]. In the case of purely brittle fracture, LEFM holds. The scaling of structural strength σ_N is self similar, described by a power law ($\sigma_N \propto D^{-1/2}$). However, for quasibrittle fracture, the structural strength scaling exhibits incomplete self-similarity, necessitating the introduction of a characteristic length scale. For geometrically scaled structures with a pre-existing crack or a notch, this size effect is well described by Bažant's size effect law [1],

$$\sigma_N = Bf'_t\left(1+\frac{D}{D_{0m}}\right)^{-1/2} \tag{2}$$

where σ_N is the nominal structure strength, D is the structure size, B is a material parameter, f'_t is a measure of the material strength, and D_{0m} is the transitional size which delineates ductile and brittle failures under monotonic loading fracture and depends on the material characteristic length as well as structural geometry. Based on asymptotic expansion of the deviations from LEFM,

$$B = \frac{1}{f'_t}\sqrt{\frac{E'G_f}{g'(\alpha_0)c_{fm}}};\quad D_{0m} = c_{fm}\frac{g'(\alpha_0)}{g(\alpha_0)} \tag{3}$$

where $c_{fm} \approx$ half the FPZ length during monotonic fracture, $\alpha_0 = a_0/D$, the relative initial crack length, $g(\alpha_0) = k^2(\alpha_0)$ = dimensionless energy release rate, $k(\alpha_0)$ = dimensionless stress intensity factor, which accounts for geometry effects, $g'(\alpha_0) = dg(\alpha)/d\alpha|_{\alpha=\alpha_0}$; for plane strain $E' = E/(1-\nu^2)$, E = Young's modulus, ν = Poisson ratio, G_f = fracture energy (under monotonic conditions). The size effect law has been proven to describe the scaling of monotonic strength in a wide variety of quasibrittle materials [1].

The size effect on Paris law for quasibrittle materials was first experimentally identified in [11,12] by fatigue tests of geometrically scaled three-point bend beams of normal and high strength concretes (NSC and HSC). It was found that the Paris law plots of $\log(da/dN)$ vs. $\log(\Delta K)$ for similar beams of different sizes are not coincident, but spaced apart and roughly parallel in a log–log plot, implying a size effect in the Paris law coefficient C but not the exponent m. To express the Paris law as a material property, independent of structure size, a size adjusted Paris law was proposed [11] in the form

$$\frac{da}{dN} = C\left(\frac{\Delta K}{K_{IC}}\right)^m \quad \text{where} \quad K_{IC} = K_{IF}\left(1+\frac{D_{0m}}{D}\right)^{-1/2} \tag{4}$$

which follows from Eqs. (2) and (3); here K_{IC} = size dependent fracture toughness, such that $\lim_{D\to\infty} K_{IC} = K_{IF}$. The purpose was to collapse the Paris law plots for different sizes into a unified plot.

However, using the D_{0m} value from the monotonic size effect law would not unify the plots. Therefore, D_{0m} in Eq. (3) was replaced by a similar expression for the transitional size for fatigue, $D_{0c} = c_{fc}g'(\alpha_0)/g(\alpha_0)$ in which $c_{fc} \approx$ half the size of the cyclic FPZ. The value of D_{0c} was determined iteratively, such that it would cause the three distinct Paris law plots for different sizes to merge into one. This confirms Rice's proposition [18] that the cyclic and monotonic FPZs must be different in size. Further it reveals approximate equality of the ratio, R_f, of the cyclic and monotonic FPZ sizes, L_{fc} and L_{fm} to the ratios of the transitional sizes and of the approximate FPZ half-sizes, i.e.,

$$R_f = \frac{D_{0c}}{D_{0m}} = \frac{c_{fc}}{c_{fm}} \approx \frac{L_{fc}}{L_{fm}} \tag{5}$$

The ratio R_f was reported as 10 in [11] but later discounted, due an error found in copying the compliance equation from the handbook. Further, R_f was reported in [12] to be ≈1.5 but after a revised fitting of the monotonic size effect law, it is now found to be ≈0.9. So, although it was originally inferred that the cyclic FPZ would be more elongated than the monotonic one, this inference now comes under question.

Recently, this fatigue size effect was studied by Le et al. [13] for sandstone, a rock that is also quasibrittle. They realized that since K_{IC} is the monotonic fracture toughness, its scaling cannot involve D_{0c}. So, they introduced a new physical quantity, viz. the fatigue fracture energy U_c, which is the energy required to propagate a fatigue crack of unit width by a unit length. The size effect in U_c involved D_{0c} and was expressed as [13],

$$U_c = U_{c,\infty}\frac{D}{D+D_{0c}} \tag{6}$$

where $U_{c,\infty}$ is the fatigue fracture energy for $D\to\infty$. Rather than normalizing ΔK by K_{IC}, Le et al. introduced the dimensionless parameter $\Delta K^2/EU_c$ and conducted dimensional analysis to reach the following size-adjusted Paris law:

$$\frac{da}{dN} = C\left(1+\frac{D_{0c}}{D}\right)^{m/2}(\Delta K)^m = C(\Delta K_D)^m \tag{7}$$

where $\Delta K_D = \Delta K(1+D_{0c}/D)^{1/2}$ = size adjusted ΔK. Interestingly, they arrived at the same form as in [12] but by using more rigorous