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A fatigue life estimation algorithm based on Statistical Energy Analysis in high-frequency random processes

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ABSTRACT

A frequency-domain fatigue life estimation algorithm based on Statistical Energy Analysis (SEA) is proposed in this study for a structure subject to high-frequency loading. The main contribution is to observe that, when evaluated at 1/3-octave bands, the RMS value of the power spectral density (PSD) function is sufficiently refined to produce meaningful fatigue life estimates. A practical application concerning the fatigue life of a plate of aircraft subject to high-frequency random loading is presented to confirm the applicability of the proposed algorithm.

1. Introduction

A structure subject to high-frequency random loading, which is commonly encountered in engineering, may cause serious fatigue damage or even breakdown in some extreme cases. For instance, the hypersonic vehicle is under complex fluctuating pressure produced by shock-induced turbulent boundary layer separation in flight [1], and the induced structural fatigue and equipment failure may cause serious damage to the vehicle. For the random loading due to fluid/structure interaction at high frequency, the application of deterministic numerical methods, such as the Finite Element Method (FEM) or the Boundary Element Method (BEM), is often restricted [2]. Statistical Energy Analysis (SEA) is a reliable tool for vibro-acoustic problems in high-frequency, however, the obtained results reflect only the energy distribution of each subsystem [3,4] and cannot be used directly for structure safety assessment.

Fatigue life evaluation is an important task in the design of mechanical components subject to random loading. Two different approaches are commonly adopted to estimate the fatigue life. One is the time-domain analysis, such as the rainflow count algorithm [5]. This method is simple from the theoretical point of view, but it is often time-consuming and expensive, and thus its application is limited to a certain extent. The other is the frequency-domain analysis. Rayleigh approximation [6,7] and Dirlik's formula [8] are two widely used classical frequency-domain methods; the former is considered to be one of the most important theoretical results, while the latter commonly produces good results when compared to the time-domain analysis [9]. Recently, the frequency-domain fatigue estimation has received much attention [10–16]. However, all the adopted methods are based on stress PSD obtained by experiments or numerical simulations. As such, the outcomes of SEA, which refer to the average energy of each subsystem, cannot be applied directly to estimate the fatigue life.

In this study, Statistical Energy Analysis is combined with the frequency-domain methods and a fatigue life estimation algorithm is proposed for a structure under high-frequency random loading. Compared with previous frequency-domain methods, this algorithm does not require the conventional stress PSD in the frequency-band to predict the fatigue life, whereas in the alternative, only the stress RMS is needed. So far no one has observed the errors that may occur if the SEA results are used for fatigue life estimation. Our new treatment is based on the careful observation of the influence of the stress PSD shape on the fatigue damage when the stress RMS in the frequency-band remains unchanged. A large number of confidential simulations shows that the fatigue damage is insensitive to the shape of the stress PSD when the bandwidth is properly divided. This conclusion can also be partly confirmed by the bands method newly proposed by Braccesi et al. [17], where the zero order moment of the PSD in each subdivided frequency-band is used to estimate the fatigue damage. Even more, through confidential simulations the upper-bound errors are obtained for the fatigue damage evaluation by replacing the real PSD with the simplified one defined in this work. In summary, under a proper bandwidth division, the conventional stress PSD is not necessary

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in the fatigue analysis, while the stress RMS in each frequency-band is sufficient for this fatigue life prediction.

2. SEA for vibro-acoustic analysis

Some difficulties are encountered by FEM and BEM in solving high-frequency problems. First, the computational cost is generally prohibitive because of the large number of degrees of freedom of the discrete system. Second, the system response becomes increasingly sensitive to geometrical imperfections. SEA is a reliable method for high-frequency structural acoustic analysis and has been widely used in aviation and aerospace engineering. Here, energy is the primary variable of interest, and other dynamic variables, such as displacement and stress, are calculated from the energy of vibration. The energy balance equation is given by [4]

$$[L]\{E\} = \frac{1}{2\pi f}\{P_{in}\} \tag{1}$$

where L is a matrix formed by the coupling loss factors and the internal loss factors, P_{in} is the input power, E is the average energy of subsystem. The equation is usually solved in one-third octave bands and f is the center frequency. The one-third octave bands for the frequency range from 891 Hz to 8985 Hz are shown in Table 1, the ratio of the upper band limit to the lower band limit is $2^{(1/3)}$ (about 1.26). After the structural model is built and the input power is calculated, the average energy of the subsystem in each one-third octave band can be obtained.

The most widely used subsystem in SEA is the plate. If the thickness of the plate is uniform and the material is isotropic, then the mean square stress can be calculated by the following formulation:

$$\langle\sigma^2\rangle = 3Y_0E/V \tag{2}$$

where Y_0 is the elasticity modulus and V is the volume of the plate.

The results obtained from SEA are the average response of the subsystem. A response concentration factor R_s is considered to denote the maximum stress, which is the stress at the dangerous point in fatigue analysis. The expression is given by

$$R_s = \psi_{\max}\sqrt{N_0} \tag{3}$$

where N_0 is the number of modes of the plate in a one-third octave band, which can be obtained from SEA and $\psi_{\max} = 2^{D/2}$ with D being the dimension of the subsystem.

The stress RMS at the dangerous point then becomes

$$\sigma_M = R_s\sqrt{\frac{3Y_0E}{V}} \tag{4}$$

3. Frequency-domain methods for fatigue damage estimation

The frequency-domain methods introduced in this section will be used in the numerical simulations throughout the paper.

3.1. Rayleigh approximation and its correction

The Rayleigh approximation method [6,7] is based on Bendat theory. In this method, the probability density function (PDF) of the stress amplitude S has a Rayleigh peak distribution and depends only on m_0 (m_i is the ith order moment, i.e., $m_i = \int \omega^i G(\omega)d\omega$, in which $G(\omega)$ is the stress PSD):

$$p_{Ray}(S) = \frac{S}{m_0}e^{-\frac{S^2}{2m_0}} \tag{5}$$

The expected damage under the Palmgren–Miner rule is

$$E[D]_{Ray} = E[0]\frac{T}{C}\int_0^\infty S^k p_{Ray}(S)dS \tag{6}$$

which depends on the S–N relation $S^kN = C$, the loading time T, and $E[0] = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}}$.

The method is suited in addressing narrow-band processes and is considered to give over-conservative results for broad-band processes. Some researchers have proposed correction formulas to reduce its damage value, such as Wirsching and Light [18]:

$$E[D]_{Wir} = E[D]_{Ray}\cdot f_{Wir} \tag{7}$$

where f_{Wir} is an empirical correction factor dependent on the S–N slope k and the bandwidth parameter α_2:

$$f_{Wir} = a(k) + [1 - a(k)](1-\xi)^{b(k)} \tag{8}$$

where $a(k) = 0.926 - 0.033k$, $b(k) = 1.587k - 2.323$, $\xi = \sqrt{1-\alpha_2^2}$, and $\alpha_2 = m_2/\sqrt{m_0\cdot m_4}$.

3.2. Dirlik's formula

Dirlik's formula [8] is a result of a best fit over a large number of data from numerical simulations. It is one of the most widely used fatigue life prediction methods and is approximated as the sum of an exponential and two Rayleigh probability densities:

$$p_{Dir}(S) = \frac{1}{\sqrt{m_0}}\left[\frac{D_1}{Q}e^{-\frac{Z}{Q}} + \frac{D_2Z}{R^2}e^{-\frac{Z^2}{2R^2}} + D_3Ze^{-\frac{Z^2}{2}}\right] \tag{9}$$

where

$$\begin{aligned} &Z = S/\sqrt{m_0}, \quad D_1 = \frac{2(x_m-\alpha_2^2)}{1+\alpha_2^2}, \quad D_2 = \frac{1-\alpha_2-D_1+D_1^2}{1-R} \\ &D_3 = 1-D_1-D_2, \qquad Q = \frac{1.25(\alpha_2-D_3-D_2R)}{D_1} \\ &R = \frac{\alpha_2-x_m-D_1^2}{1-\alpha_2-D_1+D_1^2}, \quad x_m = \alpha_1\cdot\alpha_2 = \frac{m_1}{\sqrt{m_0m_2}}\cdot\frac{m_2}{\sqrt{m_0m_4}} \end{aligned} \tag{10}$$

The bandwidth parameters α_1 and α_2 are dimensionless numbers, with $0 < \alpha_2 \leqslant \alpha_1 \leqslant 1$; in a narrow-band process they approach 1, whereas in a broad-band process they tend to 0 [19].

The expected damage is then expressed as

$$E[D]_{Dir} = E[P]\frac{T}{C}\int_0^\infty S^k p_{Dir}(S)dS \tag{11}$$

where $E[P] = \frac{1}{2\pi}\sqrt{\frac{m_4}{m_2}}$.

Table 1
Frequency limits for one-third octave bands (Hz).

Center frequency	1000	1250	1600	2000	2500	3150	4000	5000	6300	8000
Lower band limit	891	1123	1414	1782	2245	2829	3565	4492	5660	7131
Upper band limit	1123	1414	1782	2245	2829	3565	4492	5660	7131	8985