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An embedded cohesive crack model for finite element analysis of quasi-brittle materials

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ABSTRACT

This paper presents a numerical implementation of the cohesive crack model for the analysis of quasibrittle materials based on the strong discontinuity approach in the framework of the finite element method. A simple central force model is used for the stress versus crack opening curve. The additional degrees of freedom defining the crack opening are determined at the crack level, thus avoiding the need for performing a static condensation at the element level. The need for a tracking algorithm is avoided by using a consistent procedure for the selection of the separated nodes. Such a model is then implemented into a commercial program by means of a user subroutine, consequently being contrasted with the experimental results. The model takes into account the anisotropy of the material. Numerical simulations of well-known experiments are presented to show the ability of the proposed model to simulate the fracture of quasibrittle materials such as mortar, concrete and masonry.

1. Introduction

Numerical implementation of quasi-brittle cohesive cracking remains an open issue, 30 years after the introduction of the fictitious crack model by Hillerborg [1]. Traditionally, the numerical methods based on the Finite Element Method (FEM) were classified into two groups [2]: smeared crack and discrete crack approaches, although some authors include a third group: the lattice approach [3].

In the smeared crack approach the fracture is represented in a smeared manner: an infinite number of parallel cracks of infinitely small opening are (theoretically) distributed (smeared) over the finite element [4]. The cracks are usually modelled on a fixed finite element mesh. Their propagation is simulated by the reduction of the stiffness and strength of the material. The constitutive laws, defined by stress–strain relations, are non-linear and show a *strain softening*. This approach was pioneered with fixed-crack orthotropic secant models [5–7] and rotating crack models [8–10]. More elaborate models have also been proposed [11,12].

However, such *strain softening* introduces some difficulties in the analysis. The system of equations may become ill-posed [13–15], localisation instabilities and spurious mesh sensitivity of finite element calculations may appear [4]. These difficulties can be addressed by supplementing the material model with some mathematical condition [16–18]. Other strategies are

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Nomenclature

a	finite element node index
A	finite element area
$\mathbf{b}_a(x)$	shape function gradient for node a
$\mathbf{E}$	elastic moduli tensor
$f(w)$	classical softening function for mode I
f_t	tensile strength
f_{t1}	tensile strength in the material axis **1** (bed joints direction)
f_{t2}	tensile strength in the material axis **2** (head joints direction)
G_F	specific fracture energy
h	triangular element height
$H(x)$	Heaviside jump function
L	crack length in the finite element
$\mathbf{n}$	unit normal vector
$N_a(\boldsymbol{x})$	traditional shape function for node a
$\mathbf{t}$	traction vector
$\mathbf{u}_a$	nodal displacement
w	crack opening
$\mathbf{w}$	crack displacement vector
$\tilde{w}$	equivalent crack opening
α	angle between the material axis **1** (bed joints direction) and the OX axis
β	angle between the first principal stress direction and the OX axis
γ	angle between the crack direction and the material axis **1** (bed joints direction)
$\boldsymbol{\varepsilon}^c$	continuous part of the strain tensor
$\boldsymbol{\varepsilon}^a$	apparent part of the strain tensor
$\boldsymbol{\sigma}$	stress vector, with components $(\sigma_x, \sigma_y, \tau_{xy})$
θ	angle between first principal stress direction and the material axis **1** (bed joints direction)
σ_I	first principal stress
σ_γ	normal stress to the arbitrary direction (which forms an angle γ with the material axis **1**)
1	direction of the bed joints of masonry
2	direction of the head joints of masonry
I	first principal stress direction
II	second principal stress direction

Abbreviations

CMOD	crack mouth opening displacement
CMSD	crack mouth sliding displacement
EAS	enhanced assumed strain method
FE	finite element
FEM	finite element method
SDA	strong discontinuity approach
TPB	three point bending

the non-local continuum models [19,20], the gradient models [21], and the micropolar continuum [22]. These procedures are suited to specific problems, but none gives a general solution to the problem.

The discrete approach is preferred when there is one crack, or a finite number of cracks, in the structure. The cohesive crack model, developed by Hillerborg and co-authors [1] for mode I fracture of concrete, was shown to be efficient to model the fracture process of quasi-brittle materials. It has been extended to mixed mode fracture (modes I and II) and incorporated into finite element programs [23–28] and into boundary element codes [29]. One of the difficulties associated with these programs is that they require the remeshing and/or refinement of the finite element mesh when the crack grows, and some of them also require an input of material properties that are difficult to evaluate.

In recent years, a new methodology based on the so-called *strong discontinuity approach* (SDA) has been proposed [30,31]. The SDA complements the classical approaches, the smeared crack and the discrete crack, and has been successful in the analysis of the fracture of quasibrittle materials. In contrast to the smeared crack model, in the SDA the fracture zone is represented as a discontinuous displacement surface. Different from the discrete crack approach, in the SDA the crack geometry