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A size-dependent Kirchhoff micro-plate model based on strain gradient elasticity theory

Binglei Wang [a,b], Shenjie Zhou [a,*], Junfeng Zhao [a], Xi Chen [b,c,d,**]

[a] School of Mechanical Engineering, Shandong University, Jinan, Shandong 250061, China
[b] Department of Earth and Environmental Engineering, Columbia University, New York, NY 10027, USA
[c] Department of Civil & Environmental Engineering, Hanyang University, Seoul 133-791, South Korea
[d] School of Aerospace, Xi'an Jiaotong University, Xi'an 710049, China

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ABSTRACT

A size-dependent Kirchhoff micro-plate model is developed based on the strain gradient elasticity theory. The model contains three material length scale parameters, which may effectively capture the size effect. The model can also degenerate into the modified couple stress plate model or the classical plate model, if two or all of the material length scale parameters are taken to be zero. The static bending, instability and free vibration problems of a rectangular micro-plate with all edges simple supported are carried out to illustrate the applicability of the present size-dependent model. The results are compared with the reduced models. The present model can predict prominent size-dependent normalized stiffness, buckling load, and natural frequency with the reduction of structural size, especially when the plate thickness is on the same order of the material length scale parameter.

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1. Introduction

Recent technological developments have opened up promising research opportunities and engineering priorities in micro-plate based micromechanics (Batra et al., 2007), in which the plate thickness is typically on the order of microns or sub-microns. The size-dependent behavior of micron-scale structures has been proven experimentally in metals (Nix, 1989; Fleck et al., 1994; Poole et al., 1996), geomaterials and brittle materials (Vardoulakis et al., 1998), polymers (Lam and Chong, 1999; Lam et al., 2003; McFarland and Colton, 2005) and polysilicon (Chasiotis and Knauss, 2003). The classical theory of linear elasticity is characterized by the local character of stress without any internal (material) length scale, which is inadequate for predicting the mechanical behavior of small material structures, whose behavior is characterized by non-local stresses and the existence of an internal length scale.

Higher-order continuum theories have recently raised the interest of many scientists (Batra, 1987; Fleck et al., 1994; Vardoulakis et al., 1998; Lam et al., 2003; Papargyri-Beskou et al., 2003, 2010; Reddy, 2007a; Papargyri-Beskou and Beskos, 2008; Kong et al., 2009; Wang et al., 2010), in which strain gradient or non-local terms are involved and additional material length scale parameters are consequently introduced to complement the classical material constants. A review of the high order elasticity theories can be found in the works of (Vardoulakis and Sulem, 1995; Exadaktylos and Vardoulakis, 2001; Papargyri-Beskou and Beskos, 2008).

Based on the aforementioned higher-order continuum theories, several micro-plate models have been developed by many researchers based on micropolar theory (Ariman, 1968a,b); the simplest version of the simplified form-II theory of strain gradient linear elasticity due to Mindlin (1964) (Papargyri-Beskou and Beskos, 2009; Vavva et al., 2009; Papargyri-Beskou et al., 2010); gradient elastic theory (Lazopoulos, 2004, 2009); and couple stress theory (Hoffman, 1964; Ellis and Smith, 1967; Tsiatas, 2009). Ariman (1968a,b) studied the circular micropolar plate and discussed some problems in the model. Lazopoulos (2004) established a strain gradient elasticity theory of plates, based on the gradient elasticity theory proposed by Altan and Aifantis (1997) which can be traced back to Mindlin (1965). The theory is applied to the study of the buckling behavior of a long rectangular plate under uniaxial compression and small lateral load, supported on a rigid plane foundation. Recently, Lazopoulos (2009) studied the bending of strain gradient elastic thin plates, adopting a simple version of Mindlin's linear theory of elasticity with microstructure, in which

* Corresponding author.
** Corresponding author. Department of Earth and Environmental Engineering, Columbia University, New York, NY 10027, USA.
E-mail addresses: zhousj@sdu.edu.cn (S. Zhou), xichen@columbia.edu (X. Chen).

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the intrinsic bulk length g and the directional surface energy length l_k are introduced to characterize the strain gradient in addition to the classical Lame constants. Tsiatas (2009) presented a micro Kirchhoff plate model for the static analysis of isotropic micro-plates with arbitrary shape based on the simplified couple stress theory of Yang et al. (2002) containing only one material length scale parameter, rendering a relatively simple formulation of the size-dependent plate model. Vavva et al. (2009) studied the velocity dispersion curves of guided modes propagating in an isotropic micro-plate based on the simplified Mindlin (1964, 1965) form-II gradient elastic theory. Very recently, Papargyri-Beskou et al. (2010) studied the gradient elastic flexural Kirchhoff plates under static loading via variational method, and derived the exact boundary condition for any plate form and showed validated the effectiveness of the approximate boundary conditions proposed by Papargyri-Beskou and Beskos (2008).

Shu and Fleck (1998) pointed out that the couple stress theory (Fleck and Hutchinson, 1993), which is a general form of the modified couple stress theory (Yang et al., 2002) used by Tsiatas (2009) to predict the size effect of micro-plate, usually under-predicts the size effect because the couple stress theory only employs the rotation gradient and neglects the other gradients (e.g. stretch gradient). Therefore, to more effectively account for the size effect, a general strain gradient theory, incorporating not only the rotation gradient but also stretch gradient or other gradients, should be introduced.

Among the higher-order continuum theories, the strain gradient elasticity theory proposed by Lam et al. (2003) was successfully applied to predict the size-dependent properties for small scale structures. Three material length scale parameters are introduced to characterize the dilatation gradient tensor, the deviatoric stretch gradient tensor, and the symmetric rotation gradient tensor, respectively. Through work conjugation, the higher-order stress tensors are related to the higher-order deformation metrics. The theory has been used to analyze the static and dynamic problems of micro scale Bernoulli–Euler beam (Kong et al., 2009) and Timoshenko beam (Wang et al., 2010). Moreover, it should be noted that strain gradient elasticity theory of Lam et al. (2003) can degenerate into the modified couple stress theory of Yang et al. (2002) by setting two of the three material length scale parameters to zero; thus, the strain gradient elasticity theory (Lam et al., 2003) may be regarded as a much wider extension of the modified couple stress theory (Yang et al., 2002).

The objective of this work is to develop a size-dependent Kirchhoff plate model based on the strain gradient elasticity theory (Lam et al., 2003). In Section 2, the governing equation of the size-dependent Kirchhoff micro-plate is derived. In subsequent Sections 3–5, the size-dependence of the normalized stiffness, critical load, and natural frequency for the simple supported plate are described and discussed. Conclusions are summarized in Section 6.

2. Governing equations of size-dependent flexural plate

Based on the higher-order stress theory (Mindlin, 1965), Lam et al. (2003) proposed the strain gradient elasticity theory, in which a new additional equilibrium equation governing the behavior of higher-order stresses, the equilibrium of moments of couples, is introduced in addition to the classical equilibrium equations of forces and moments. There are three material length scale parameters for isotropic linear elastic materials.

According to the theory, the total deformation energy density is a function of the symmetric strain tensor, the dilatation gradient vector, the deviatoric stretch gradient tensor and the symmetric rotation gradient tensor. The strain energy U in a deformed isotropic linear elastic material occupying region Ψ (with a volume element V) is given by

$$U = \frac{1}{2}\int_V \bar{u}\,\mathrm{d}\Psi = \frac{1}{2}\iiint_V \bar{u}\,\mathrm{d}x\mathrm{d}y\mathrm{d}z \tag{1}$$

in which $\bar{u}$ is the strain energy density, defined by

$$\bar{u} = \sigma_{ij}\varepsilon_{ij} + p_i\gamma_i + \tau_{ijk}^{(1)}\eta_{ijk}^{(1)} + m_{ij}^s\chi_{ij}^s \tag{2}$$

For the indices (subscripts) throughout this paper, the repeated indices denote summation from 1 to 3. And the deformation measures, i.e., the strain tensor, ε_{ij}, the dilatation gradient tensor, γ_i, the deviatoric stretch gradient tensor, $\eta_{ijk}^{(1)}$, and the symmetric rotation gradient tensor, χ_{ij}^{s}, are defined by

$$\varepsilon_{ij} = \frac{1}{2}(\partial_j u_i + \partial_i u_j) \tag{3}$$

$$\eta_{ijk}^{(1)} = \eta_{ijk}^s - \frac{1}{5}\left(\delta_{ij}\eta_{mmk}^s + \delta_{jk}\eta_{mmi}^s + \delta_{ki}\eta_{mmj}^s\right) \tag{4}$$

$$\gamma_i = \partial_i\varepsilon_{mm} \tag{5}$$

and

$$\chi_{ij}^s = \frac{1}{4}(e_{ipq}\partial_p\varepsilon_{qj} + e_{jpq}\partial_p\varepsilon_{qi}) \tag{6}$$

respectively. Here, ∂_i is the differential operator, u_i is the displacement vector, ε_{mm} is the dilatation strain, and η_{ijk}^{s} is the symmetric part of the second order displacement gradient tensor defined by

$$\eta_{ijk}^s = \frac{1}{3}\left(u_{i,jk} + u_{j,ki} + u_{k,ij}\right) \tag{7}$$

where δ_{ij} and e_{ijk} are the Knocker delta and permutation tensor, respectively.

The stress measures (detailed physical interpretation of the higher-order stresses can be found in Lam et al. (2003)) include the classical stress tensor, σ_{ij}, and the higher-order stresses, p_i, $\tau_{ijk}^{(1)}$, and m_{ij}^{s}, which are the work-conjugate to the deformation measures, are given by the following constitutive relations,

$$\sigma_{ij} = k\delta_{ij}\varepsilon_{mm} + 2\mu\varepsilon'_{ij} \tag{8}$$

$$p_i = 2\mu l_0^2\gamma_i \tag{9}$$

$$\tau_{ijk}^{(1)} = 2\mu l_1^2\eta_{ijk}^{(1)} \tag{10}$$

$$m_{ij}^s = 2\mu l_2^2\chi_{ij}^s \tag{11}$$

where ε_{ij}' is the deviatoric strain defined as

$$\varepsilon'_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_{mm}\delta_{ij} \tag{12}$$

k and μ are the bulk and shear modulus, respectively. l_0, l_1 and l_2 are the additional independent material length scale parameters associated with the dilatation gradients, deviatoric stretch gradients, and symmetric rotation gradients, respectively.

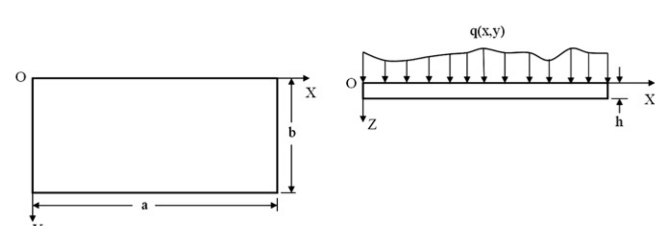

Fig. 1. Schematic of a micro-plate with distributed load.

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