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Technical note

A fixed point in the Coffin–Manson law

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ABSTRACT

It is assumed that the Coffin–Manson law exhibits a fixed point at varying fatigue ductility exponent for given series of tests. This provides for the law three constants, the bilinearity characterised by a change of fixed point and exponent and, at stage I and through a length scale, the expressions of the constants of a kinetic equation of the crack. The results are addressed to observations in austenitic stainless steels of the literature and discussed in term of microstructures determining the type and propagation mechanism of the crack measured by the fixed point and exponent, respectively.

1. Introduction

The Coffin–Manson law relates for a metal uniaxially and cyclically loaded the plastic strain amplitude ε_p to the cycle number of fracture N_f through a two-parameter power law

$$\varepsilon_p = \varepsilon_f (N_f)^c \tag{1}$$

where ε_f and c (<0) are the fatigue ductility coefficient and exponent, respectively [1,2]. A fatigue test exhibiting the Coffin–Manson law will be defined by the metal (chemical composition and pretreatment), the conditions (temperature and medium) and the range of values of ε_p investigated. The specimens will be assumed of smooth surface and, up to Section 7, their geometries and dimensions not relevant for ε_f and c. This law has been little discussed [3]. Interpretations have been proposed in terms of cyclic strain hardening exponent, for example [4], of statistical nature of the surface damage [5], of slip [6] or of coefficients of a kinetic equation of the crack [7].

It was shown by correlation that ε_f and c can be connected through a linear $log(\varepsilon_f)-c$ relationship for diverse series of tests [6,8], but this is no proof of its existence. If assumed valid, its coefficients have potential to be related to the damage by fatigue [8]. This was applied to the Paris law, with evidence of a fixed point for the law [9], as well as to creep rate and rupture time [10].

The article will assume the existence of the linear $log(\varepsilon_f)-c$ relationship after discussion of experimental results for low cycle fatigue. It will present consequences for the law, including a fixed point at varying c, form, constants and bilinearity, as well as, for stage I of fatigue through a length scale, the propagation rate of the fatal crack (crack) associated to the law and the expressions of the constants of the above-mentioned kinetic equation with the fixed point and c. Results will be addressed to observations of the literature for austenitic stainless steels and discussed in terms of two microstructures associated to the type and propagation mechanism of the crack and measured by the fixed point and c, respectively.

2. Existence of a fixed point for the Coffin–Manson law

The normalisation of N_f by N_f^I allows to rewrite Eq. (1) [10]

$$\varepsilon_p = \varepsilon_f \left(N_f^I\right)^c \left(N_f/N_f^I\right)^c = \varepsilon_p^I \left(N_f/N_f^I\right)^c \tag{2}$$

where ε_p^I is a constant associated to N_f^I through ε_f and c which normalises ε_p. Eq. (2) is the reduced form of the Coffin–Manson law with three constants, N_f^I, ε_p^I and c. One has (Eqs. (1) and (2))

$$\varepsilon_f = \varepsilon_p^I \left(N_f^I\right)^{-c} \tag{3}$$

Eq. (2) lets open the possibility of ε_p^I non depending on c. In this case, N_f^I and ε_p^I are the coordinates of a fixed point of Eq. (2) at varying c, which is unique. This yields the linear $log(\varepsilon_f)-c$ relationship (Eq. (3))

$$log(\varepsilon_f) = log\left(\varepsilon_p^I\right) - c\,log\left(N_f^I\right) \tag{4}$$

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The determination of ε_p^I and N_f^I requires several tests assuming their physical origin and values unchanged as well as different values of ε_f and c, with their statistical errors of measurement, fulfilling Eq. (4) through correlation.

Three cases will be briefly discussed with the preliminary assumption of the existence of a fixed point for each of them. Eq. (4) was reliably put in evidence for 88 unalloyed steels at 23 °C with an error of margin of about 1% and confidence level of 99% for the correlation coefficients [6]. However, the plot exhibits a relatively large number of points situated relevantly below the correlation line within about a band parallel to it. The cause, measurement error or microstructure, is uncertain, but a different fixed point cannot be ruled out for the corresponding steels. At 23 and 300 °C, the $log(\varepsilon_f)-c$ plots of these 88 steels are parallel and very near, suggesting the existence of a range of temperatures above 23 °C with the same fixed point. For 17 austenitic stainless steels fatigued at 23 °C, taken from data of ASM [11] and assumed to be stable, Eq. (4) is obtained with the correlation coefficient of 0.993 (Fig. 1). The steels include types 321 (STQ), 321 (STA), 316 (STQ), 347 (STQ) and 304 (STQ), STQ and STA denoting the pretreatment: solution treated, quenched for STQ and solution treated, aged or annealed for STA. The same fixed point for these steels is not ensured due to the small number of steels of each type and distributions of $(c, log(\varepsilon_f))$ in the plot for certain of them.

These examples confirm that the existence of a fixed point for a series of tests fulfilling Eq. (4), even with good statistical reliability, is an open question. The fulfilment of Eq. (4) can be only assumed, this through good statistical reliability including also the statistical errors of measurement of ε_f and c, inspection of the $log(\varepsilon_f)-c$ plot and physical support.

In this article, it will be assumed that the series of tests or particular tests referred exhibit a fixed point, which can differ according to the series or test. It will be often denoted by I or $\left(N_f^I, \varepsilon_p^I\right)$. For the steels of ASM, one has for N_f^I 6606 within, from the standard deviation, [5128, 8511] and for ε_p^I 0.347% within [0.309%, 0.389%]. The fixed point is shown in Fig. 2 for the plots of four of them calculated with Eqs. (1) and (4). Since the estimate of the range of values of ε_f and c scanned by a given type of steel requires a large number of tests [12], far beyond the seventeen considered here, the present value of $\left(N_f^I, \varepsilon_f^I\right)$ cannot be extended to the general case of the austenitic stainless steels. But it will be used as reference to illustrate certain results of this article.

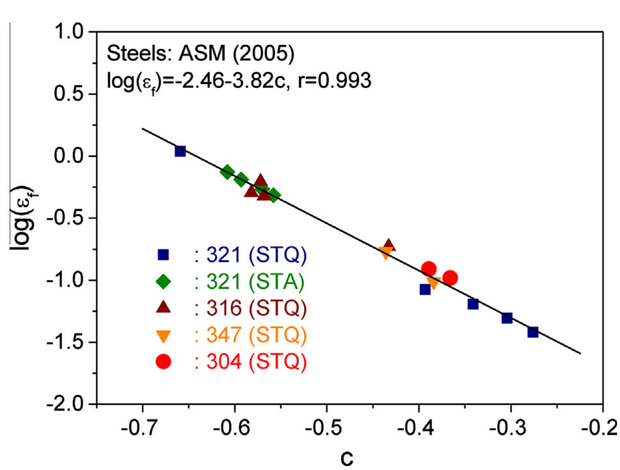

Fig. 1. $log(\varepsilon_f)$ versus c for the steels of ASM. 321 (STQ): square; 321 (STA): rhombus; 316 (STQ): triangle; 347 (STQ): inversed triangle; 304 (STQ): circle.

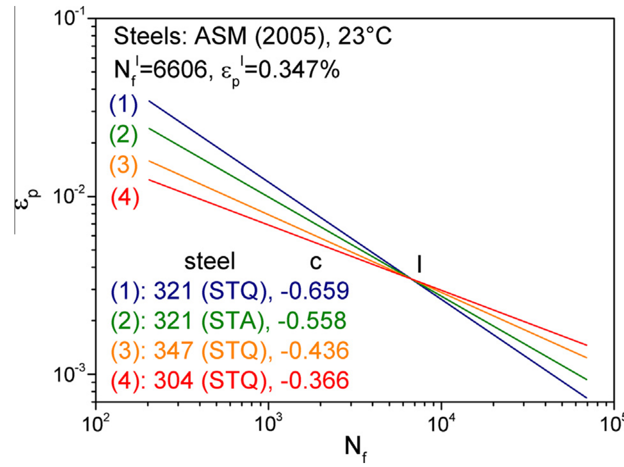

Fig. 2. Coffin–Manson plots of four steels of ASM intersecting at the fixed point.

3. N_f versus c and Coffin–Manson bilinear plot in term of fixed point

One has for N_f (Eq. (2))

$$N_f = N_f^I\left(\varepsilon_p/\varepsilon_p^I\right)^{1/c} \tag{5}$$

Cycle N_f versus c (Eq. (5), Fig. 3) shows that, given ε_p, N_f increases or decreases as a function of c according to ε_p smaller or larger than ε_p^I, respectively. This result was already mentioned from intersecting Coffin–Manson plots in pure copper at three different temperatures [13]. It is due to the exponential function of N_f versus ε_p going through fixed point $\left(\varepsilon_p^I, N_f^I\right)$ at any c.

The bilinear plot is characterised by two subsequent branches with a change of values of c to c' and of ε_f to ε_f' at the transition point $(N_{ft}, \varepsilon_{pt})$. It was interpreted in terms of elastic and inelastic components of the strain amplitude [14] or of change of active slip systems observed in the volume of the specimen and fracture mode [15]. It can also be related to the fixed point. By defining the branches by $\left(N_f^I, \varepsilon_p^I\right)$ and $\left(N_f^{I'}, \varepsilon_p^{I'}\right)$, one obtains (Eq. (5))

$$N_f^{I'} = N_f^I\left(\varepsilon_p^I\right)^{-1/c}\left(\varepsilon_{pt}\right)^{(c'-c)/c'c}\left(\varepsilon_p^{I'}\right)^{1/c'} \tag{6}$$

Eq. (6) shows that the case $c' = c$ is not consistent with the fact that the fixed point does not depend on the exponent. Therefore the case of two collinear branches with different fixed points,

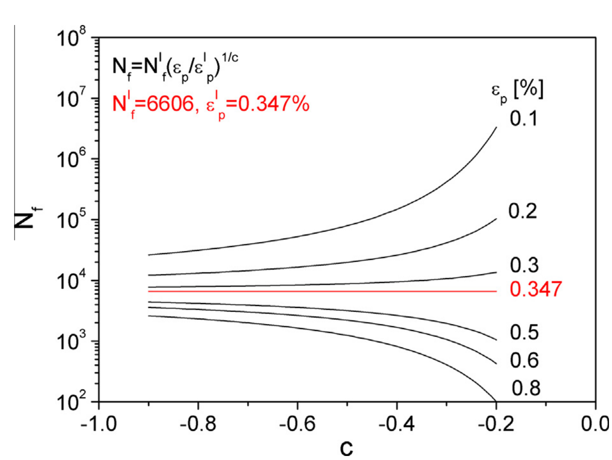

Fig. 3. N_f versus c calculated for diverse ε_p with the fixed point of the steels of ASM.