International Journal of Fatigue 78 (2015) 1–10

Contents lists available at ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

A new approach of fatigue life prediction for metallic materials under multiaxial loading

B.C. Li [a], C. Jiang [a,*], X. Han [a], Y. Li [b]

[a] State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, College of Mechanical and Vehicle Engineering, Hunan University, Changsha City 410082, China
[b] Department of Traffic and Transportation Engineering, College of Basic Education for Commanding Officers, National University of Defense Technology, Changsha 410082, China

ARTICLE INFO

Article history:
Received 28 October 2014
Received in revised form 31 January 2015
Accepted 22 February 2015
Available online 12 March 2015

Keywords:
Multiaxial fatigue
Equal-life curve
Life prediction
New path-dependent factor
Material sensitivity

ABSTRACT

Based on experimental data found in literatures, four traditionally multiaxial fatigue life criteria are analyzed and verified. It is discovered that these conventional criteria cannot reflect well the combined effect both under tension and torsion loadings for some materials, such as 6082-T6 and AlCu4Mg1, due to lack of enough consideration about the influence of stress amplitude ratio and stress level on fatigue life even under proportional loading. In order to solve this problem, a new approach of fatigue life prediction, based on the equal-life curve, is proposed and it is composed of three parts: the multiaxial fatigue life surface, a new path-dependent factor for multiaxial high-cycle fatigue and a material parameter describing material sensitivity to non-proportional loading. Finally, the precision of the presented approach is systematically checked against the experimental data found in literatures for four different materials under proportional and non-proportional loadings.

1. Introduction

Although fatigue is regarded as the main cause of many mechanical failures, fatigue mechanisms are still not fully understood. This is partly due to the complexity of geometrical shape and complex loading of components and structures, which result in multiaxial stress state. Actually, with the development of modern industry, many critical components, such as axles, crankshafts and turbine disks, are always subjected to multiaxial fatigue loading during their service life.

A significant amount of research has been devoted in the past few decades to acquire a better understanding of the failure mechanisms under multiaxial loading paths. Based on different failure mechanisms, various methods that can be broadly classified into three main approaches have been proposed. Firstly, the earliest works, which are called equivalent stress method, on multiaxial stress states are usually based on extensions of static yield theories to fatigue under combined stresses, such as the classical yield theories of Lame and Tresca in the late 19th century and von Mises in the early 20th century [1]. Specifically, the Von-Mises criterion appears to be the most popular one as it defines the relationship between axial and shear stress components as: $\sigma = \sqrt{3}\tau$, which is very convenient for engineering design. However, the predicted results are always non-conservative under non-proportional loading [2]. Secondly, the use of stress invariant and hydrostatic stress as correlating parameters for multiaxial fatigue is another approach to the prediction of multiaxial fatigue life. In Crossland [3] and Sines et al. [4] invariant criteria, the combined damage, from axial and shear stress components, is calculated through the maximum amplitude of the second deviatoric invariant and the hydrostatic stress. Lastly, critical plane method, such as Matake [5] and Papadopoulos criteria [6,7], appears to be the most successful method for multiaxial fatigue life prediction. The main advantage of this method is that it makes agreement with the physical observation that a fatigue crack usually initiates on a particular plane. In general, the critical plane method is usually composed of two steps: the first step is to find the critical plane which may be the maximum shear stress plane, maximum normal stress plane or the characteristic plane related to fracture plane; The second step is to use the characteristic parameters, such as normal and shear stress amplitude on the critical plane, to constitute particular formula and finally compute the damage degree. Despite the popularity achieved by this method, it still has some shortcomings. For example, the same stress amplitude is acquired at different planes, meaning several critical planes; nevertheless, the fractographic examination generally indicates only one initiation spot.

Although there are numerous methods predicting multiaxial fatigue life, in fact, no matter how different these criteria are, the

* Corresponding author. Tel.: +86 731 88821748; fax: +86 731 88821445.
E-mail address: jiangc@hnu.edu.cn (C. Jiang).

http://dx.doi.org/10.1016/j.ijfatigue.2015.02.022
0142-1123/

core idea of them all aims at building up the relationship between the damage from the axial and shear stress components, and furthermore quantifying this relationship. The one main difference among these criteria, from authors' point of view, is related to the number of given conditions. More specifically, before predicting the multiaxial fatigue limit, the equivalent stress method such as Von-Mises criterion only needs the fatigue limit in fully reserved axial loading σ_{-1}, while almost other criteria, such as Gough [8,9], Matake and Papadopoulos criteria, require both the fatigue limit in fully reserved axial loading σ_{-1} and fatigue limit in fully reserved torsion τ_{-1}. Meanwhile, the combined effect under tension and torsion in Von-Mises criterion is quantified by 0.577 independence of material, whereas the latter always regards τ_{-1}/σ_{-1} as an important parameter to compute the combined damage degree. Nevertheless, the corresponding experiments [10–12] have demonstrated that the loading path may cause a strong influence on the material fatigue strength, namely, multiaxial fatigue life will be affected both by the stress amplitude ratio and stress level. However, almost all criteria mentioned above obviously seems not to consider enough these two factors due to that material constant τ_{-1}/σ_{-1} or 0.577 is rarely related to stress level and stress amplitude ratio and litter consideration of these two factors may lead to large prediction error.

In the present study, the inadequacy of four conventional criteria is firstly analyzed, and then a new approach of multiaxial fatigue life prediction for metallic materials, based on equal-life curve, is presented by taking into account the influence of stress amplitude ratio and stress level as well as material sensitivity to non-proportional loading. Lastly, in order to verify the capability of this new approach, experimental data from four kinds of materials are used and satisfactory results are obtained.

2. Stress state under multiaxial cycle loading

The multiaxial fatigue stress state under tension and torsion loading is shown in Fig. 1. If the applied loading is sine wave:

$$\sigma_{xx}(t) = \sigma_a \sin wt \tag{1}$$

$$\tau_{xy}(t) = \tau_a \sin(wt - \delta) \tag{2}$$

$$\lambda = \tau_a/\sigma_a \tag{3}$$

where σ_a and τ_a are the axial and shear stress amplitudes, respectively; δ is the phase angle and λ is the stress amplitude ratio.

The normal stress σ_α and the shear stress τ_α on the plane which makes an angle α with the X-axis can be expressed as:

$$\sigma_\alpha = \sigma_{xx}\cos^2\alpha + 2\tau_{xy}\sin\alpha\cos\alpha \tag{4}$$

$$\tau_\alpha = \tau_{xy}\cos 2\alpha - \sigma_{xx}\sin\alpha\cos\alpha \tag{5}$$

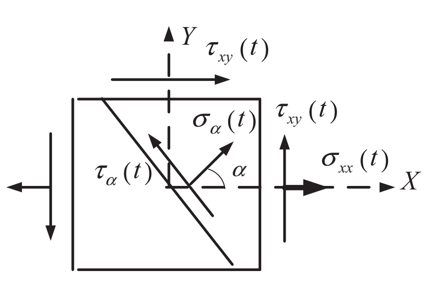

Fig. 1. Multiaxial fatigue stress state.

where σ_α and τ_α are the functions of time t and angle α, therefore, the normal stress amplitude $\sigma_{a,\alpha}$ and shear stress amplitude $\tau_{a,\alpha}$ in a cycle on the plane α can be given by:

$$\sigma_{a,\alpha} = \frac{\max_t(\sigma_\alpha(t)) - \min_t(\sigma_\alpha(t))}{2} \tag{6}$$

$$\tau_{a,\alpha} = \frac{\max_t(\tau_\alpha(t)) - \min_t(\tau_\alpha(t))}{2} \tag{7}$$

$$\tau_{a,\min} = \min_\alpha(\tau_{a,\alpha}) \tag{8}$$

where $\tau_{a,\min}$ is the minimum shear stress amplitude in all planes during a loading cycle and it is convenient to calculate the $\sigma_{a,\alpha}$ and $\tau_{a,\alpha}$ no matter whether the applied loading is sine wave or not by using Eqs. (6) and (7).

3. Multiaxial fatigue life criteria

3.1. Von-Mises criterion

In elastic mechanics field, the Von-Mises criterion is based on assumption that only shear stress causes plastic flow and the hydrostatic stress has no effect on the yield process. In order to estimate fatigue life, Von-Mises criterion can be written as follows [13]:

$$\sigma_{eq,vm}(t) = \sqrt{\sigma_{xx}^2(t) + 3\tau_{xy}^2(t)} \tag{9}$$

$$\sigma_{eq} = \frac{\max_t[\sigma_{eq,vm}sgn(I_1)] - \min_t[\sigma_{eq,vm}sgn(I_1)]}{2} = f_1(N) \tag{10}$$

where $\sigma_{eq,vm}$ and σ_{eq} are the equivalent stress and the equivalent stress amplitude, respectively. I_1 is the first stress tensor invariant during loading. $f_1(N)$ is the uniaxial S–N curve. It is obvious that Eq. (10) can be degenerated into uniaxial form in the case of uniaxial loading. The geometrical meaning of this criterion is described clearly in Fig. 2. As long as the loading path with zero mean stress can be enveloped in the same ellipse, the predicted life will be same no matter whether the loading is proportional or non-proportional one. It might also be noted that the ellipse curve is the equal-life curve of Von-Mises criterion.

3.2. Gough criterion

Gough criterion is composed of two equations for metals under combined in-phase bending and torsion. The first equation, namely the ellipse quadrant equation for ductile metals, is given as:

$$\left(\frac{\sigma_a}{\sigma_{-1}}\right)^2 + \left(\frac{\tau_a}{\tau_{-1}}\right)^2 = 1 \tag{11}$$

The second equation, called ellipse arc equation for brittle metals, is given as:

$$\left(\frac{\tau_a}{\tau_{-1}}\right)^2 + \left(\frac{\sigma_a}{\sigma_{-1}}\right)^2\left(\frac{\sigma_{-1}}{\sigma_{-1}} - 1\right) + \left(\frac{\sigma_a}{\sigma_{-1}}\right)\left(2 - \frac{\sigma_{-1}}{\sigma_{-1}}\right) = 1 \tag{12}$$

Both equations can also be transformed into uniaxial form and pure torsion form in the case of uniaxial and pure torsion loading, respectively. Meanwhile, two different equal-life curves can be acquired at the same time.

3.3. Matake criterion

This criterion is a typical critical plane method and the critical plane is regarded as the plane experiencing the maximum shear stress amplitude. The criterion takes the following form: