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Thin plate spline radial basis functions for vibration analysis of clamped laminated composite plates

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ABSTRACT

A meshless method based on thin plate spline radial basis functions and higher-order shear deformation theory are presented to analyze the free vibration of clamped laminated composite plates. The singularity of thin plate spline radial basis functions is eliminated by adding infinitesimal to the zero distance. Convergence characteristics of the present thin plate spline radial basis functions for the vibration analysis of the clamped laminated plates are investigated. The frequencies computed by the present method agree well with the available published results.

1. Introduction

Laminated composite plates have been widely used in industries, especially in aerospace due to their high strength-to-weight ratio. With the wide application of laminated composite plates, vibration analysis of laminated composite plates becomes an important task (Khdeir and Reddy, 1999; Khdeir and Librescu, 1988; Wang, 1997; Aagaah et al., 2006; Akhras, 2005; Baharlou and Leissa, 1987; Shi et al., 2004; Reddy and Phan, 1985; Soldatos and Messina, 2001).

The meshless method in which the problem domain is represented by a set of distributed nodes have been successfully applied to analyze the free vibration of laminated composite plates. Liew et al. (2004) used the reproducing kernel particle method and FSDT to analyze the free vibration and buckling of shear-deformable plates. FSDT and the moving least squares differential quadrature method were applied to vibration analysis of symmetrically laminated plates by Liew et al. (2003). Chen et al. (2003) used the element free Galerkin method for the free vibration analysis of composite laminates of complicated shape. Wu et al. (2005) applied the moving least squares differential quadrature method to analyze the vibration of generally laminated composite plates. The multiquadrics radial basis functions were applied to analyze the free vibration of laminated composite plates by Ferreira (2005), Ferreira and Fasshauer (2006), Ferreira et al. (2005) and Roque et al. (2006). The inverse multiquadrics radial basis functions were used to analyze the free vibration of laminated composite plates by Ferreira and Fasshauer (2007) and Xiang and Wang (2009). Gaussian radial basis functions were used to analyze the free vibration of laminated composite plates by Xiang et al. (2009). The compact support Wendland radial basis functions were used to analyze the free vibration of composite and sandwich plates by Ferreira et al. (2008).

The multiquadric, inverse multiquadric and Gaussian radial basis functions include a shape parameter which have important effect on the accuracy. The choice of shape parameter has been an intense subject (Wang and Liu, 2002; Carlson and Foley, 1991). Thin plate spline doesn't need the shape parameter, but it has the disadvantage of singularity when the distance between node i and node j is zero.

In this paper, the singularity of thin plate spline radial basis function is eliminated by adding infinitesimal to the zero distance. The main objective of the present paper is to demonstrate the meshless method based on thin plate spline radial basis functions can be successfully used to analyze the free vibration of clamped laminated composite plates. The numerical examples show that the frequencies computed by the present method agree well with the available published results.

2. The radial basis function method

Radial basis functions method is a truly meshless method which approximates the whole solution of the partial differential

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equations using radial basis functions. Radial basis functions method was used to solving partial differential equations by Kansa (1990a,b). Most-widely used radial basis functions are

$$\text{Multiquadric } g_j = \sqrt{r_{ij}^2 + c^2}$$

$$\text{Inverse multiquadric } g_j = 1\Big/\sqrt{r_{ij}^2 + c^2}$$

$$\text{Gaussian } g_j = e^{-cr_{ij}^2}$$

$$\text{Thin plate spline } g_j = r_{ij}^{2m}\log(r_{ij}) \quad m = 1,2,3\ldots$$

where $r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$ denotes the distance between node (x_i, y_i) and node (x_j, y_j). c is the shape parameter.

Radial basis functions method for solving partial differential equations is based on a scattered data interpolation problem. The solution of partial differential equations is approximated by radial basis functions in the form of

$$U = \sum_{j=1}^{N} \alpha_j g_j \tag{1}$$

where N is the total number of nodes, α_j is unknown coefficients, g_j is radial basis function. Radial basis function used in this paper is thin plate spline as follows

$$g_j = r_{ij}^6 \log(r_{ij})$$

In order to eliminate the singularity of thin plate spline, $r_{ij} = r_{ij} + \varsigma$ when $r_{ij} = 0$.

where ς is infinitesimal.

3. Differential governing equations based on higher-order shear deformation theories

Differential governing equations based on Higher-order shear deformation theories (Reddy and Phan, 1985; Reddy, 1984) are

$$\begin{aligned}
&A_{11}\frac{\partial^2 u}{\partial x^2} + A_{12}\frac{\partial^2 v}{\partial x\partial y} + A_{16}\left(\frac{\partial^2 u}{\partial x\partial y} + \frac{\partial^2 v}{\partial x^2}\right) + B_{11}\frac{\partial^2 \phi_x}{\partial x^2} + B_{12}\frac{\partial^2 \phi_y}{\partial x\partial y} + B_{16}\left(\frac{\partial^2 \phi_x}{\partial x\partial y} + \frac{\partial^2 \phi_y}{\partial x^2}\right) - C_1E_{11}\left(\frac{\partial^2 \phi_x}{\partial x^2} + \frac{\partial^3 w}{\partial x^3}\right) - C_1E_{12}\left(\frac{\partial^2 \phi_y}{\partial x\partial y} + \frac{\partial^3 w}{\partial x\partial y^2}\right)\\
&-C_1E_{16}\left(\frac{\partial^2 \phi_x}{\partial x\partial y} + \frac{\partial^2 \phi_y}{\partial x^2} + 2\frac{\partial^3 w}{\partial x^2\partial y}\right) + A_{16}\frac{\partial^2 u}{\partial x\partial y} + A_{26}\frac{\partial^2 v}{\partial y^2} + A_{66}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x\partial y}\right) + B_{16}\frac{\partial^2 \phi_x}{\partial x\partial y} + B_{26}\frac{\partial^2 \phi_y}{\partial y^2} + B_{66}\left(\frac{\partial^2 \phi_x}{\partial y^2} + \frac{\partial^2 \phi_y}{\partial x\partial y}\right)\\
&-C_1E_{16}\left(\frac{\partial^2 \phi_x}{\partial x\partial y} + \frac{\partial^3 w}{\partial x^2\partial y}\right) - C_1E_{26}\left(\frac{\partial^2 \phi_y}{\partial y^2} + \frac{\partial^3 w}{\partial y^3}\right) - C_1E_{66}\left(\frac{\partial^2 \phi_x}{\partial y^2} + \frac{\partial^2 \phi_y}{\partial x\partial y} + 2\frac{\partial^3 w}{\partial x\partial y^2}\right) = I_1\frac{\partial^2 u}{\partial t^2} + \bar{I}_2\frac{\partial^2 \phi_x}{\partial t^2} - C_1I_4\frac{\partial^2}{\partial t^2}\left(\frac{\partial w}{\partial x}\right)
\end{aligned} \tag{2}$$

$$\begin{aligned}
&A_{16}\frac{\partial^2 u}{\partial x^2} + A_{26}\frac{\partial^2 v}{\partial x\partial y} + A_{66}\left(\frac{\partial^2 u}{\partial x\partial y} + \frac{\partial^2 v}{\partial x^2}\right) + B_{16}\frac{\partial^2 \phi_x}{\partial x^2} + B_{26}\frac{\partial^2 \phi_y}{\partial x\partial y} + B_{66}\left(\frac{\partial^2 \phi_x}{\partial x\partial y} + \frac{\partial^2 \phi_y}{\partial x^2}\right) - C_1E_{16}\left(\frac{\partial^2 \phi_x}{\partial x^2} + \frac{\partial^3 w}{\partial x^3}\right) - C_1E_{26}\left(\frac{\partial^2 \phi_y}{\partial x\partial y} + \frac{\partial^3 w}{\partial x\partial y^2}\right)\\
&-C_1E_{66}\left(\frac{\partial^2 \phi_x}{\partial x\partial y} + \frac{\partial^2 \phi_y}{\partial x^2} + 2\frac{\partial^3 w}{\partial x^2\partial y}\right) + A_{12}\frac{\partial^2 u}{\partial x\partial y} + A_{22}\frac{\partial^2 v}{\partial y^2} + A_{26}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x\partial y}\right) + B_{12}\frac{\partial^2 \phi_x}{\partial x\partial y} + B_{22}\frac{\partial^2 \phi_y}{\partial y^2} + B_{26}\left(\frac{\partial^2 \phi_x}{\partial y^2} + \frac{\partial^2 \phi_y}{\partial x\partial y}\right)\\
&-C_1E_{12}\left(\frac{\partial^2 \phi_x}{\partial x\partial y} + \frac{\partial^3 w}{\partial x^2\partial y}\right) - C_1E_{22}\left(\frac{\partial^2 \phi_y}{\partial y^2} + \frac{\partial^3 w}{\partial y^3}\right) - C_1E_{26}\left(\frac{\partial^2 \phi_x}{\partial y^2} + \frac{\partial^2 \phi_y}{\partial x\partial y} + 2\frac{\partial^3 w}{\partial x\partial y^2}\right) = I_1\frac{\partial^2 v}{\partial t^2} + \bar{I}_2\frac{\partial^2 \phi_y}{\partial t^2} - C_1I_4\frac{\partial^2}{\partial t^2}\left(\frac{\partial w}{\partial y}\right)
\end{aligned} \tag{3}$$

$$\begin{aligned}
&A_{45}\left(\frac{\partial \phi_y}{\partial x} + \frac{\partial^2 w}{\partial x\partial y}\right) + A_{55}\left(\frac{\partial \phi_x}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) - 3C_1D_{45}\left(\frac{\partial \phi_y}{\partial x} + \frac{\partial^2 w}{\partial x\partial y}\right) - 3C_1D_{55}\left(\frac{\partial \phi_x}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + A_{44}\left(\frac{\partial \phi_y}{\partial y} + \frac{\partial^2 w}{\partial y^2}\right) + A_{45}\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial^2 w}{\partial x\partial y}\right)\\
&-3C_1D_{44}\left(\frac{\partial \phi_y}{\partial y} + \frac{\partial^2 w}{\partial y^2}\right) - 3C_1D_{45}\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial^2 w}{\partial x\partial y}\right) - 3C_1\left[D_{45}\left(\frac{\partial \phi_y}{\partial x} + \frac{\partial^2 w}{\partial x\partial y}\right) + D_{55}\left(\frac{\partial \phi_x}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) - 3C_1F_{45}\left(\frac{\partial \phi_y}{\partial x} + \frac{\partial^2 w}{\partial x\partial y}\right) - 3C_1F_{55}\right.\\
&\times\left(\frac{\partial \phi_x}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right) + D_{44}\left(\frac{\partial \phi_y}{\partial y} + \frac{\partial^2 w}{\partial y^2}\right) + D_{45}\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial^2 w}{\partial x\partial y}\right) - 3C_1F_{44}\left(\frac{\partial \phi_y}{\partial y} + \frac{\partial^2 w}{\partial y^2}\right) - \left.3C_1F_{45}\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial^2 w}{\partial x\partial y}\right)\right] + C_1\left[E_{11}\frac{\partial^3 u}{\partial x^3} + E_{12}\frac{\partial^3 v}{\partial x^2\partial y}\right.\\
&+E_{16}\left(\frac{\partial^3 u}{\partial x^2\partial y} + \frac{\partial^3 v}{\partial x^3}\right) + F_{11}\frac{\partial^3 \phi_x}{\partial x^3} + F_{12}\frac{\partial^3 \phi_y}{\partial x^2\partial y} + F_{16}\left(\frac{\partial^3 \phi_x}{\partial x^2\partial y} + \frac{\partial^3 \phi_y}{\partial x^3}\right) - C_1H_{11}\left(\frac{\partial^3 \phi_x}{\partial x^3} + \frac{\partial^4 w}{\partial x^4}\right) - C_1H_{12}\left(\frac{\partial^3 \phi_y}{\partial x^2\partial y} + \frac{\partial^4 w}{\partial x^2\partial y^2}\right) - C_1H_{16}\\
&\times\left(\frac{\partial^3 \phi_x}{\partial x^2\partial y} + \frac{\partial^3 \phi_y}{\partial x^3} + 2\frac{\partial^4 w}{\partial x^3\partial y}\right) + E_{12}\frac{\partial^3 u}{\partial x\partial y^2} + E_{22}\frac{\partial^3 v}{\partial y^3} + E_{26}\left(\frac{\partial^3 u}{\partial y^3} + \frac{\partial^3 v}{\partial x\partial y^2}\right) + F_{12}\frac{\partial^3 \phi_x}{\partial x\partial y^2} + F_{22}\frac{\partial^3 \phi_y}{\partial y^3} + F_{26}\left(\frac{\partial^3 \phi_x}{\partial y^3} + \frac{\partial^3 \phi_y}{\partial x\partial y^2}\right) - C_1H_{12}\\
&\times\left(\frac{\partial^3 \phi_x}{\partial x\partial y^2} + \frac{\partial^4 w}{\partial x^2\partial y^2}\right) - C_1H_{22}\left(\frac{\partial^3 \phi_y}{\partial y^3} + \frac{\partial^4 w}{\partial y^4}\right) - C_1H_{26}\left(\frac{\partial^3 \phi_x}{\partial y^3} + \frac{\partial^3 \phi_y}{\partial x\partial y^2} + 2\frac{\partial^4 w}{\partial x\partial y^3}\right) + 2E_{16}\frac{\partial^3 u}{\partial x^2\partial y} + 2E_{26}\frac{\partial^3 v}{\partial x\partial y^2} + 2E_{66}\left(\frac{\partial^3 u}{\partial x\partial y^2} + \frac{\partial^3 v}{\partial x^2\partial y}\right)\\
&+2F_{16}\frac{\partial^3 \phi_x}{\partial x^2\partial y} + 2F_{26}\frac{\partial^3 \phi_y}{\partial x\partial y^2} + 2F_{66}\left(\frac{\partial^3 \phi_x}{\partial x\partial y^2} + \frac{\partial^3 \phi_y}{\partial x^2\partial y}\right) - 2C_1H_{16}\left(\frac{\partial^3 \phi_x}{\partial x^2\partial y} + \frac{\partial^4 w}{\partial x^3\partial y}\right) - 2C_1H_{26}\left(\frac{\partial^3 \phi_y}{\partial x\partial y^2} + \frac{\partial^4 w}{\partial x\partial y^3}\right) - 2C_1H_{66}\\
&\times\left.\left(\frac{\partial^3 \phi_x}{\partial x\partial y^2} + \frac{\partial^3 \phi_y}{\partial x^2\partial y} + 2\frac{\partial^4 w}{\partial x^2\partial y^2}\right)\right] = I_1\frac{\partial^2 w}{\partial t^2} - C_1^2I_7\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + C_1I_4\frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + C_1\bar{I}_5\frac{\partial^2}{\partial t^2}\left(\frac{\partial \phi_x}{\partial x} + \frac{\partial \phi_y}{\partial y}\right)
\end{aligned} \tag{4}$$