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Reflection of plane waves at the free surface of a monoclinic thermoelastic solid half-space

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ABSTRACT

Lord—Shulman and Green—Lindsay theories of generalized thermoelasticity are applied to study the reflection from a thermally insulated stress-free thermoelastic solid half-space of monoclinic type. A particular model is chosen for the numerical computations of reflection coefficients. Effects of anisotropy and relaxation times are observed on reflection coefficients.

1. Introduction

Thermoelasticity deals with the dynamical systems whose interactions with surroundings include not only mechanical work and external work but also exchange of heat. Biot (1956) explained thermoelasticity by deriving dilatation based on the thermodynamics of irreversible process and coupling it with elastic deformation. But the diffusion type heat equation used in this study predicted infinite speed for propagation of thermal signals. Lord and Shulman (1967) defined the generalized theory of thermoelasticity in which a hyperbolic equation of heat conduction with a relaxation time ensured the finite speed for thermal signals. Using two relaxation times, Green and Lindsay (1972) developed another generalized theory of thermoelasticity. A unified treatment of both Lord and Shulman and Green and Lindsay theories was presented by Ignaczak and Ostoja-Starzewski (2009). Dhaliwal and Sherief (1980) extended Lord and Shulman (1967) generalization of thermoelasticity for anisotropic case. Chandrasekhariah (1986) presented a review of work done in the theory of thermoelasticity.

The thermoelasticity has wide applications in various fields such as earthquake engineering, soil dynamics, aeronautics, astronautics, nuclear reactors, high energy particle accelerator, etc. Thermoelasticity is also used in polymer coating and to evaluate the stress redistribution in ceramic matrix composites (Mackin and Purcell, 1996; Barone and Patterson, 1998). The study of wave propagation in a generalized thermoelastic media with additional parameters like anisotropy, porosity, viscosity, microstructure, temperature and other parameters provide vital information about existence of new or modified waves. Such information may be useful for experimental seismologists in correcting earthquake estimation.

There are reasonable grounds for assuming anisotropy in the continents. Seismic anisotropy, now widely known as a common feature of most subsurface formations, may lead to significant distortions in conventional seismic processing, such as errors in velocity analysis, mispositioning of reflectors, and misinterpretation of the amplitude variation with offset (AVO) response.

Investigation of waves in anisotropic materials are considerably more difficult than the classical and well-understood, isotropic problem. Isotropic materials can be characterized by only two parameters; anisotropic materials can have from 5 to 21 independent material constants. In addition to the increased number of parameters, the geometry of the body and symmetry of the material may complicate the analysis. In an isotropic solid, the behavior of the material is the same in all directions. Anisotropic solids show a preference to certain directions. Reconciliation of the geometry of the body with these preferred material directions constitutes a large part of analysis in many anisotropic problems.

In an anisotropic elastic solid medium, three types of body waves with mutually orthogonal particle motion can be propagated. In general, the particle motion is neither purely longitudinal nor purely transverse. Because of this, the three types of body waves in an anisotropic medium are referred to as qP, qSV and qSH

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rather than as P SV and SH the symbols used for propagation in an isotropic medium (Keith and Crampin, 1977). Chattopadhyay and Choudhury (1995) discussed the reflection of qP waves at the plane free boundary of a monoclinic half-space. In a subsequent paper, Chattopadhyay et al. (1996) studied the reflection of qSV waves. Singh (1999) commented on above two papers, where the authors assume that qP waves are purely longitudinal and qSV waves purely transverse. He found that most of the results of these two papers, including the expressions for the reflection coefficients, are erroneous. Singh and Khurana (2002) studied the propagation of plane waves in an anisotropic elastic medium possessing monoclinic symmetry. The expressions for the phase velocity of qP and qSV waves propagating in the plane of elastic symmetry are obtained in terms of the direction cosines of the propagation vector. They have shown that, in general, qP waves are not longitudinal and qSV waves are not transverse. Pure longitudinal and pure transverse waves can propagate only in certain specific directions.

The problems on wave propagation in isotropic and anisotropic thermoelastic solids are studied by many authors. Prominent among them are Chadwick and Sneddon (1958), Deresiewicz (1960), Flavin (1962), Chadwick and Seet (1970), McCarthy (1972), Puri (1973), Banerjee and Pao (1974), Sinha and Sinha (1974), Chadwick (1979), Sharma and Sidhu (1986), Sharma (1988), Sinha and Elsibai (1996, 1997), Singh and Kumar (1998), Verma (2002), Abd-alla et al. (2003), Sharma et al. (2003), Singh (2003, 2006), Othman and Song (2007) and Singh (2008).

Singh (2006) studied the plane wave propagation in a monoclinic generalized thermoelastic medium with thermal relaxations and has shown the existence of three plane quasi waves, namely, quasi-thermal (qT) wave, quasi-P (qP) wave and quasi-SV (qSV) wave in a two-dimensional model of monoclinic generalized thermoelastic medium in context of Lord and Shulman (1967) and Green and Lindsay (1972) theories. In the present paper, reflection of these plane waves from stress-free thermally insulated surface of a monoclinic thermoelastic solid half-space is studied. Reflection coefficients of various reflected waves are obtained and studied numerically for a particular model to analyze effects of anisotropy and thermal relaxations.

2. Governing equations of generalized thermoelasticity of monoclinic type

Consider a homogeneous, anisotropic, generalized thermoelastic medium of monoclinic type at a uniform temperature. The origin is taken on the thermally insulated and stress-free plane surface and z-axis is directed normally into the half-space which is represented by $z \geq 0$. Following Lord and Shulman (1967) and Green and Lindsay (1972), Singh (2006) derived the governing field equations of generalized monoclinic thermoelasticity for two-dimensional motion in the y–z plane ($\partial/\partial x \equiv 0$,) and in the absence of body forces and heat sources as

$$c_{22}\frac{\partial^2 u_2}{\partial y^2} + c_{44}\frac{\partial^2 u_2}{\partial z^2} + c_{24}\frac{\partial^2 u_3}{\partial y^2} + c_{34}\frac{\partial^2 u_3}{\partial z^2} + 2c_{24}\frac{\partial^2 u_2}{\partial y \partial z} + (c_{23}+c_{44})\frac{\partial^2 u_3}{\partial y \partial z} - \beta_2\frac{\partial}{\partial y}\left(T + t_1\frac{\partial T}{\partial t}\right) = \rho\frac{\partial^2 u_2}{\partial t^2}, \quad (1)$$

$$c_{24}\frac{\partial^2 u_2}{\partial y^2} + c_{34}\frac{\partial^2 u_2}{\partial z^2} + c_{44}\frac{\partial^2 u_3}{\partial y^2} + c_{33}\frac{\partial^2 u_3}{\partial z^2} + 2c_{34}\frac{\partial^2 u_3}{\partial y \partial z} + (c_{23}+c_{44})\frac{\partial^2 u_2}{\partial y \partial z} - \beta_3\frac{\partial}{\partial z}\left(T + t_1\frac{\partial T}{\partial t}\right) = \rho\frac{\partial^2 u_3}{\partial t^2}, \quad (2)$$

$$K_2\frac{\partial^2 T}{\partial y^2} + K_3\frac{\partial^2 T}{\partial z^2} - T_0\left[\beta_2\left(\frac{\partial^2 u_2}{\partial y \partial t} + t_0\Omega\frac{\partial^3 u_2}{\partial y \partial t^2}\right) + \beta_3\left(\frac{\partial^2 u_3}{\partial z \partial t} + t_0\Omega\frac{\partial^3 u_3}{\partial z \partial t^2}\right)\right] = \rho C_e\left(\frac{\partial T}{\partial t} + t_0\frac{\partial^2 T}{\partial t^2}\right), \quad (3)$$

where

$$\beta_2 = (c_{12}+c_{22})\alpha_2 + c_{23}\alpha_3,\ \beta_3 = 2c_{23}\alpha_2 + c_{33}\alpha_3, \quad (4)$$

and u_2, u_3 are components of displacement vector $\mathbf{u}$ in y–z plane, T is change in temperature above the reference non-uniform temperature T_0, ρ is the density of medium, C_e is specific heat at constant strain, c_{ij} are the isothermal elasticities; t_0, t_1 are thermal relaxation times; K_2, K_3 and α_2, α_3 are thermal conductivities and the coefficients of linear thermal expansion along and perpendicular to the axis of symmetry. The use of symbol Ω in Eq. (3) makes these fundamental equations possible for two different theories of the generalized thermoelasticity. For the L–S (Lord–Shulman) theory $t_1 = 0$, $\Omega = 1$ and for G–L (Green–Lindsay) theory $t_1 > 0$, $\Omega = 0$. The thermal relaxations t_0 and t_1 satisfy the inequality $t_1 \geq t_0 \geq 0$ for the G–L theory only.

3. Propagation of plane waves

The solutions of Eqs. (1)–(3) are now sought in the form of the harmonic travelling wave

$$\{u_2, u_3, T\} = \{A, B, C\}e^{\iota k(ct - p_2 y - p_3 z)}, \quad (5)$$

where k is the wave number, c is the phase speed, (p_2, p_3) are the components of propagation vector and A, B, C are arbitrary constants.

Making use of (5) in Eqs. (1)–(3) and eliminating A, B and C, we obtain the following cubic equation

$$\zeta^3 + L\zeta^2 + M\zeta + N = 0, \quad (6)$$

Here,

$$\zeta = \rho c^2,$$

$$L = -\left[D_1\tau^* + D_2\tau^* + L_3 + \epsilon\eta\left(\overline{\beta}^2 p_3^2 + p_2^2\right)\right]\Big/\tau^*,$$

$$M = -\Big(-D_1D_2\tau^* - D_1L_3 - D_2L_3 - D_1\epsilon\eta\overline{\beta}^2 p_3^2 + L_1^2\tau^* + 2L_1\epsilon\eta\overline{\beta}p_2p_3 - D_2\epsilon\eta p_2^2\Big)\Big/\tau^*,$$

$$N = -(D_1D_2L_3 - L_1^2L_3)/\tau^*,$$

where

$$D_1 = c_{22}p_2^2 + c_{44}p_3^2 + 2c_{24}p_2p_3,\ D_2 = c_{44}p_2^2 + c_{33}p_3^2 + 2c_{34}p_2p_3,$$

$$L_1 = c_{24}p_2^2 + c_{34}p_3^2 + (c_{23}+c_{44})p_2p_3,\ L_2 = K_2p_2^2 + K_3p_3^2,$$

$$L_3 = \frac{L_2}{C_e},\ \overline{\beta} = \frac{\beta_3}{\beta_2},\ \epsilon = \frac{\beta_2^2 T_0}{\rho C_e v_1^2},$$

$$\eta = \tau\tau' v_1^2,\ \tau = t_0\Omega - (i/\omega),\ \tau' = 1 + i\omega t_1,\ v_1^2 = c_{22}/\rho.$$