Engineering Fracture Mechanics 99 (2013) 295–305

Contents lists available at SciVerse ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech

A theoretical method for determining initiation toughness based on experimental peak load

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ARTICLE INFO

Article history:
Received 10 May 2012
Received in revised form 24 January 2013
Accepted 25 January 2013

Keywords:
Concrete
Initiation toughness
Theoretical method
Peak load
Tensile softening curve

ABSTRACT

A theoretical method for determining the initiation toughness was proposed using input conditions of the peak load and the tensile softening curve. In this method, the cohesive stress is considered to calculate the values of critical effective crack length, crack tip opening displacement, and the initiation toughness. The calculated results using the proposed method were compared with those by the double-*K* method+. The comparison showed that the values obtained by the proposed method were slightly greater than those by the double-*K* method. Moreover, the calculation results of initiation toughness by the proposed method were verified to be independent of the specimen size and not sensitive to the tensile softening curve.

1. Introduction

A large number of studies on concrete fracture have shown that a crack grows nonlinearly before the peak load is reached, and a fracture process zone exists at the tip of a crack [1–4]. Many fracture models have been proposed to consider the fracture process zone in determining fracture parameters, such as the fictitious crack model [5], size effect model [6,7], effective crack model [8,9], two-parameter fracture model [2,10], double-*K* model [3], among others.

Concrete crack begins to grow nonlinearly when the external load reaches a critical level. The load that characterizes the initiation of crack growth is generally defined as the initial cracking load [2,11]. Similarly, the stress intensity factor corresponding to the initial cracking load is termed as the inherent toughness, i.e., initiation toughness K_{IC}^{ini} [3]. K_{IC}^{ini} resists the propagation of a crack, and a crack does not propagate until the stress intensity factor at the initial crack tip reaches K_{IC}^{ini} [12].

K_{IC}^{ini} plays a significant role in studies on concrete fracture. First, macroscopic damage in concrete is believed to initiate once the stress intensity factor at the crack tip reaches K_{IC}^{ini} [4]. At this moment, the crack propagation state changes from linear to nonlinear and, consequently, the stress state varies [2,13]. Therefore, characterizing the crack initiation state in the entire process of concrete fracture is important. Second, as expressed in Eq. (1) [12,14], K_{IC}^{ini}, is an important composition of crack extension resistance in terms of K_R-curves, which has been widely applied in describing the crack propagation in concrete structure [12,14–16].

$$K_R(\Delta\alpha) = K_{IC}^{ini} + K^{\sigma} \tag{1}$$

where $\Delta\alpha$ is the effective crack extension, and K^{σ} is the stress intensity factor caused by the cohesive stresses.

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0013-7944/$ - see front matter
http://dx.doi.org/10.1016/j.engfracmech.2013.01.012

Nomenclature

a	effective crack length
a_0	initial crack length
a_c	critical effective crack length at peak load
Δa_c	effective crack extension at peak load
B	width of specimen
c_1, c_2	material constants for nonlinear softening function
$CMOD$	crack mouth opening displacement
$CTOD$	crack tip opening displacement
$CTOD_c$	critical crack tip opening displacement
D	depth of specimen (characteristic dimension)
f_c	compressive strength of concrete
f_t	tensile strength of concrete
G_F	fracture energy
K_{IC}^{ini}	initiation toughness
K_{IC}^{un}	unstable toughness
K^{σ}	stress intensity factor due to the cohesive stress
M_1, M_2, M_3	parameters of weight function
P	external applied load
P_{ini}	initial cracking load
P_{max}	peak load
σ	cohesive stress
w_0	critical value of crack opening displacement at which the value $\sigma = 0$

Moreover, from an engineering aspect, K_{IC}^{ini} can be used as an early warning criterion for fracture during the construction and operation of structures. For instance, the criterion of K_{IC}^{ini} for concrete fracture has been recently included in the 2005 Norm for Fracture Test of Hydraulic Concrete in China [17] for concrete structures.

Considering the significance of K_{IC}^{ini} in both theoretical and practical aspects, its mechanical characteristics and calculation methods are worth studying. However, only few studies on the initiation fracture state of concrete have been conducted, and K_{IC}^{ini} is always measured using tests or calculated via empirical approaches.

Jenq and Shah [2], who argued that crack initiates once the stress intensity factor at the initial crack tip reaches half the magnitude of the critical stress intensity factor, implemented an empirical method for K_{IC}^{ini}. Similarly, Yon et al. [18] suggested that the external load at the moment of crack initiation is about 50% of the peak load. These studies shed light on the estimation of K_{IC}^{ini} but lack theoretical bases.

As for the experimental method, after measuring the initial cracking load, K_{IC}^{ini} can then be calculated using formulas based on linear elastic fracture mechanics [3]. Several measurement methods for initial cracking load, such as tests using photoelastic coating, laser speckle [3], and strain gauge [4], have been formulated. However, different test methods sometimes generate inconsistent results, and test conditions may have a significant impact on the measurement of the initial cracking load. Alternatively, as an easy way, the initial cracking load is assumed to be equal to the load corresponding to the turning point that separates the linear and nonlinear segments of the load *P–CMOD* (crack mouth opening displacement) curve obtained from tests [19,20]. The initial cracking load can be measured carefully through this method.

As discussed above, the empirical and test methods for calculating K_{IC}^{ini} are quite rough, and possible errors in the results obtained through these approaches can hardly be estimated quantitatively since the theoretical value of initial cracking load is unknown. Therefore, theoretical research on K_{IC}^{ini} is valuable. However, investigations of this aspect have been rare.

As an example of the few existing studies in this respect, the double-*K* method developed by Xu and Reinhardt [20–22] used the superposition of the unstable toughness K_{IC}^{un} and the stress intensity factor K_{IC}^{σ} caused by the cohesive stress in the effective crack tip to calculate K_{IC}^{ini} as follows:

$$K_{IC}^{ini} = K_{IC}^{un} - K_{IC}^{\sigma} \tag{2}$$

Kumar and Barai [23] recently developed a weight function method to calculate the intensity strength factor of cohesive stress, and effectively avoided the complex numerical integration commonly required in the conventional calculation, made the double-*K* method easy to apply.

The critical effective crack length α_c, and the critical crack tip opening displacement $CTOD_c$ are prerequisites for calculating K_{IC}^{un} and K_{IC}^{σ} in Eq. (2). However, the measurement of critical effective crack length in tests remains an issue because the crack length of the specimen surface is generally greater than that in the interior [11,20,22]. Moreover, Xu and Reinhardt [20], based on the linear asymptotic superposition assumption, used a linear elastic fracture mechanics and an empirical formula to calculate α_c and $CTOD_c$, respectively. Nevertheless, the nonlinear behavior of a material is known to be apparent