International Journal of Fatigue 71 (2015) 26–34

Contents lists available at ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

Sharp edged contacts subject to fretting: A description of corner behaviour

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ARTICLE INFO

Article history:
Received 28 October 2013
Received in revised form 13 February 2014
Accepted 17 February 2014
Available online 28 February 2014

Keywords:
Asymptotic approaches
Complete contacts
Fretting fatigue
Mode mixity
Williams' solution

ABSTRACT

In this paper, we use the singular terms in Williams' solution to quantify the behaviour at the edge of a *complete* (i.e. sharp edged) contact. To do this, we define two alternative parameters from the generalised stress intensity factors to bring out an internal length dimension from the solution. We then obtain an order of magnitude estimate of the extent of slip and/or separation when these remain near to the contact edge. When larger slip or separation lengths are implied, we derive only qualitative implications. Finally, we apply this analysis to an example problem.

1. Introduction

The two most commonly occurring types of contact are *incomplete* and *complete*. The former are exemplified by the Hertzian contact [1] and are usually amenable to half-plane or half-space theory. This means that the contact problem may be solved without considering what happens distant from the interface because the contact solution is substantially independent of the rest of the structure in the hinterland. Furthermore, if the bodies are elastically similar, the problem is *uncoupled*: that is, normal loads give rise only to normal tractions, and shear loads give rise only to shear tractions. Lastly, at least for the plane form of the problem, the procedure for determining the mixture of stick and slip zones that result from shear is well known. The Cattaneo solution [2] for the Hertz contact was the first of these solutions, and since then there have been many generalisations of the results and procedure [3–7].

Alas, complete contacts (i.e. sharp edged contacts) have none of these attributes; the state of stress in the neighbourhood of the contact cannot, formally, be treated independently of what arises in the rest of the body, there is a large degree of coupling, and there is no straightforward way to solve for the contact tractions or the mix of stick and slip that will arise. Thus, it is usually necessary to employ numerical methods to solve the problem, and this will give rise to the usual difficulties in attaining convergence near the contact edge, particularly if slip occurs in this region.

Our motivation for studying complete contacts comes from their practical occurrence in some engineering components, e.g. shaft splines in aero-engines, and the difficulty in accurately assessing their performance, especially when subjected to fatigue conditions. In general, the first steps in predicting a contact's performance are: (i) to identify the region of the contact interface where failure typically initiates and (ii) to determine the contact behaviour and stress state in the vicinity of this region. For complete contacts, such as the clamped cantilever shown in Fig. 1, failure commonly initiates near the sharp corner formed at the contact edge [8,9]; therefore, an accurate description of corner behaviour is required. But as it is often difficult to obtain sufficient accuracy near the contact edge using numerical methods (e.g. the finite element method), it is often best to use an asymptotic approach [10].

To determine which asymptotic form is appropriate, the behaviour at the contact edge must be known. The three most likely conditions to arise at the edge of a complete contact are shown in Fig. 2, which are: (a) closed and stuck, (b) closed and slipping (leading edge), and (c) open and slipping (trailing edge). Asymptotic solutions have already been presented in the literature that describe each of these with a high degree of accuracy. The simplest of these is of course when the contact edge region is closed and stuck (Fig. 2(a)). Under these conditions, the displacement field

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http://dx.doi.org/10.1016/j.ijfatigue.2014.02.015
0142-1123/

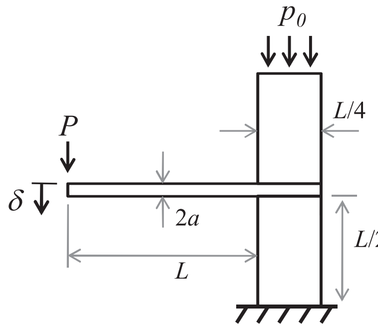

Fig. 1. An idealised diagram of a clamped cantilever test rig.

across the contact interface is continuous, so a *bilateral* model[1] can be used. When the contacting bodies are elastically similar, Williams' solution [11] for a (semi-infinite) monolithic notch is the relevant bilateral solution. If the bodies are not elastically similar, the Bogy solution [12,13] must be used instead. However, we consider only the elastically similar case here.

For the other two types of contact behaviour (Fig. 2(b and c)), the distributed dislocation technique [14] can be used to enforce the *unilateral* contact conditions. This approach enables the true extent of slip and/or separation at the contact edge to be calculated (i.e. it accounts for the redistribution of contact tractions resulting from slip and/or separation). This procedure involves performing a numerical inversion of Cauchy singular integral equations, which can be a rather involved process and must be carried out separately for each contact angle, ϕ, where ϕ is defined in Fig. 2(a). Nevertheless, this has been done for the *closed and slipping* case for contact between a quarter-plane and an elastically similar half-plane by Churchman and Hills [15]. More recently, the *open and slipping* case has also been solved for the same problem by Paynter et al. [16].

Here, we present a more modest approach for describing the behaviour at the edge of a complete contact, but which is very easy to apply and hence to use in practice. The primary output of this analysis is information on which of the three types of contact behaviour will arise, including detailed information on when transitions in behaviour will occur (e.g. from Fig. 2(a) to Fig. (c)). If slip and/or separation are implied to occur *near* the contact edge, we also obtain an order of magnitude estimate of their extent based on violations of the Signorini conditions (i.e. we do not account for the redistribution of contact tractions). When larger slip or separation extents are implied, we derive only qualitative implications. Note that although the basic formulation described in this paper is applicable to complete contacts with arbitrary edge angle, we will particularise the solution to the $\phi = 90^\circ$ case at various points to facilitate interpretation of the results. We will also frequently apply these results to the clamped cantilever shown in Fig. 1 to illustrate the deductions that can be made.

2. Williams' solution

Let us begin by considering the simplest contact behaviour, i.e. when the contact is closed and stuck as in Fig. 2(a), so we may assume the bilateral contact condition. In this situation, if we 'zoom in' arbitrarily close to the contact edge, eventually it will begin to look like a semi-infinite notch, such as that shown in Fig. 3. Thus, if the bodies are elastically similar, we can use Williams' solution [11] to describe the stress state in this region. See Barber's book [17] for an extended explanation of the form of the solution. Here we merely quote the result that the stress state in this region may be written in the form

$$\sigma_{ij}(r,\theta) = K_I r^{\lambda_I - 1} f^I_{ij}(\theta) + K_{II} r^{\lambda_{II} - 1} f^{II}_{ij}(\theta) + \text{bounded terms}, \quad (1)$$

with respect to the polar coordinate set (r,θ) shown in Fig. 3, and where $i,\ j \in \{r,\theta\}$. In this expression, the mode n generalised stress intensity factor is denoted K_n, where $n \in \{I, II\}$, and the corresponding eigenvalue and angular eigenfunctions are denoted λ_n and $f^n_{ij}(\theta)$, respectively. Finally, if we assume the plane strain condition, then

$$\sigma_{zz} = \nu(\sigma_{rr} + \sigma_{\theta\theta}), \quad (2)$$

where ν is Poisson's ratio.

Williams' solution represents the stress state at the contact edge as the superposition of two eigenfunction series expansions, which correspond to symmetric (mode *I*) and anti-symmetric (mode *II*) terms. Although both of these extend over an infinite number of terms, only the first term in each of these may imply an elastic stress singularity as $r \to 0$. Specifically, the first term in the mode *I* expansion is singular for all contacts (i.e. for $0^\circ < \phi < 180^\circ$), whereas the first term in the mode *II* expansion is singular for $77.4^\circ < \phi < 180^\circ$ and is bounded for $0^\circ < \phi < 77.4^\circ$. As we are specifically interested in the contact edge region where these singular stresses will dominate behaviour, we neglect all higher order (bounded) terms. The eigenvalues corresponding to these (potentially) singular terms are given by the lowest roots of the following equations

$$\lambda_I \sin 2\alpha + \sin 2\alpha\lambda_I = 0, \quad (3a)$$
$$\lambda_{II} \sin 2\alpha - \sin 2\alpha\lambda_{II} = 0, \quad (3b)$$

where 2α is the total included angle in the notch (see Fig. 3). Note that hereafter when we refer to the mode *I* or mode *II* eigensolutions or to Williams' solution itself, we are referring only to these two terms, i.e. the first term in the series expansion for each eigensolution.

The full expressions for the angular eigenfunctions are given in Appendix A, but here we simply note that we have normalised these such that $f^I_{\theta\theta}(0) = 1$ and $f^{II}_{r\theta}(0) = 1$. Furthermore, the angular eigenfunctions have the property that they uncouple along the bisector, i.e. that $f^I_{r\theta}(0) = 0$ and $f^{II}_{\theta\theta}(0) = 0$, so we can define the generalised stress intensity factors as

$$K_I = \lim_{r\to 0} \sigma_{\theta\theta}(r,0) r^{1-\lambda_I}, \quad (4a)$$
$$K_{II} = \lim_{r\to 0} \sigma_{r\theta}(r,0) r^{1-\lambda_{II}}. \quad (4b)$$

Note that whereas the eigenvalues, λ_n, and angular eigenfunctions, f^n_{ij}, are fully determined by the notch angle, 2α, the generalised stress intensity factors, K_I, K_{II}, depend on the finite geometry of the problem and the way the remote loads are applied; hence, they must usually be determined using numerical methods, e.g. the finite element method. One way to calculate K_I, K_{II} is to use a bilateral model of the contact geometry to determine the stress state resulting from the individual application of each applied load. These stresses can then be used with Eq. (4) to obtain a 'calibration' for the generalised stress intensity factors, i.e. the values of K_I, K_{II} resulting from each applied load. Of course, many other techniques can also be used, e.g. [18].

To model contacts, we now adopt a more convenient notation by replacing 2α with $180^\circ + \phi$. In other words, we consider contact between a sharp wedge of interior angle ϕ and a half-plane (i.e. a notch of interior angle 180°) as shown in Fig. 2(a). We emphasise that this is not an accurate representation of the overall finite geometry under consideration, e.g. the clamped cantilever in Fig. 1. Instead, it only represents the region near the contact edge

[1] Here, bilateral is meant in the sense that the interface can transmit both tension and compression, so the solution varies linearly with the applied loads. In contrast, with a unilateral model, the interface can only support compression and separates when subjected to tension.