European Journal of Mechanics A/Solids 28 (2009) 1014–1025

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

journal homepage: www.elsevier.com/locate/ejmsol

A theoretical analysis of the dynamic response of metallic sandwich beam under impulsive loading

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ARTICLE INFO

Article history:
Received 14 November 2008
Accepted 14 April 2009
Available online 22 April 2009

Keywords:
Sandwich beam
Impulsive loading
Dynamic response

ABSTRACT

The objective of this paper is to analytically study the dynamic response of a fully clamped metallic sandwich beam under impulsive loading. The membrane factor method is employed to derive the solutions for large deflections and time responses of the sandwich beam, in which the interaction of bending and stretching is considered. Moreover, tighter 'bounds' of the solutions are obtained. It is shown that the present solutions are in good agreements with the previous finite element results and lie in the bounds of the solutions. It is clear that core strength and membrane force induced by large deflections have significant effects on the dynamic response of sandwich beam with increasing the transient deflections. The present method is efficient and simple for the dynamic response analysis of large deflections of metallic sandwich structures.

1. Introduction

Lightweight structures have been widely used in a number of critical structures, such as vehicles, ships, aircrafts, and spacecrafts, etc. As a kind of new members in the family of lightweight structures, metallic sandwich beams, plates and shells with various cores have received great attention. Several kinds of metallic cores are developed, such as metallic foams, lattice materials, woven materials and egg-box, etc. (Noor et al., 1996; Gibson and Ashby, 1997; Ashby et al., 2000; Deshpande and Fleck, 2001, 2003; Sypeck and Wadley, 2001; Wadley et al., 2003). Over the past few decades, extensive works have been devoted to analyzing the dynamic responses of monolithic solid structures. Symonds (1954) carried out the small deflection analysis of the dynamic response of fully clamped monolithic solid beams subject to blast loading. Neglecting the elastic effect, Jones (1971) extended the work of Symonds (1954) and obtained an approximate solution for the dynamic response of large deflections of fully clamped monolithic solid beams. Using the approximate yield criteria, Symonds and Jones (1972) obtained the upper and lower bounds' predictions of the dynamic response of large deflections of fully clamped monolithic solid beams under blast loading. One can see more details on the main progress in the dynamic response analysis of monolithic solid structures from Jones (1989). Thereafter, Yu and Stronge (1990) proposed a membrane factor method on the basis of energy equilibrium to study the dynamic response of a rigid-perfectly plastic monolithic solid beam on a foundation, in which the effect of the membrane force induced by large deflections is considered.

Recently, some researches have been carried out to study the dynamic response of metallic sandwich structures with various kinds of cores. Qiu et al. (2003) and Xue and Hutchinson (2004) carried out the finite element calculations to investigate the dynamic responses of fully clamped metallic sandwich beams and plates subjected to impulsive loading, respectively. Fleck and Deshpande (2004) theoretically studied the shock resistance of fully clamped sandwich beams subject under uniform transverse blast loading. Subsequently, Qiu et al. (2005) proposed an analytical model for the dynamic response of fully clamped sandwich beams subjected to blast loading over a central patch. Based on the relative time-scales of compression and the combination of plastic bending and longitudinal stretching, Tilbrook et al. (2006) developed a lumped mass model and carried out finite element calculations to study the impulsive response of sandwich beams. Using a shock simulation technique involving high speed impact of aluminium foam projectiles, Rathbun et al. (2006), Radford et al. (2006) and Tagarielli et al. (2007) experimentally investigated the dynamic response of fully clamped sandwich beams. The dynamic response of metallic sandwich panels, beams and plates with different kinds of cores and subjected to underwater and air blast loadings was numerically studied by Liang et al. (2007), McShane et al. (2007), Rabeczuk et al. (2004, 2007), Hutchinson and Xue (2005) and Vaziri and Hutchinson (2007), respectively, in which fluid-structure interaction was considered.

On the other hand, Qiu et al. (2003) and Fleck and Deshpande (2004) obtained the upper and lower bounds' predictions of the

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doi:10.1016/j.euromechsol.2009.04.002

dynamic response of fully clamped metallic sandwich beams subjected to blast loading on the basis of classical yield criterion. However, the classical yield criterion may be highly accurate for the metallic sandwich structure with thin, strong face sheets and a thick, weak core. It becomes less accurate as the sandwich structures approach the limit of monolithic solid ones.

The objective of this paper is to analytically study the impulsive response of metallic sandwich beams under blast loading, in which the interaction of bending and stretching and the strength of core are considered. Moreover, tighter bounds of the solutions are derived. Also, we obtain the solutions for the dynamic response of large deflections of solid monolithic beams as a degenerate case of the sandwich beams. Comparisons of the present solutions with the previous finite element results are presented and good agreements are found.

2. Statement of the problem

Here, we consider a fully clamped metallic sandwich beam subjected to blast loading I, as shown in Fig. 1, in which $2L$, h and c are the span of the beam, the thickness of the identical face sheets and the metallic core, respectively. It is reasonable to assume that the face sheet metal and the metallic core obey the rigid-perfectly plastic laws, as shown in Fig. 2(a) and (b), respectively, in which σ_f, σ_n, σ_l, ε_D are the yield strength of face sheet metal, the normal and the longitudinal strength and the critical densification strain of metallic core, ρ_f and ρ_c are the mass density of the face sheets and the core, respectively.

The dynamic response of fully clamped metallic sandwich beams subjected to blast loading over the entire span has been studied by Fleck and Deshpande (2004), in which the deformation of core compression and beam bending and stretching are decoupled because the time period of the core compression is much smaller than overall structural response time of the sandwich structure. Moreover, in the core compression stage, they assumed a one-dimensional slice through the thickness of the sandwich beam and a quasi-static dissipation in core, and neglected the reduction in momentum due to the impulse provided by the fully clamped supports. These assumptions are adopted herein.

It is assumed that the impulse I per unit area imparts to the upper face sheet with a velocity $v_0 = I/(2\rho_f h)$. The upper face sheet compresses the core and accelerates the lower face sheet. According to the momentum conservation law, Fleck and Deshpande (2004) obtained the final common velocity v_f of the core and two face sheets at the end of core compression stage,

$$v_f = \frac{I}{2\rho_f h + \rho_c c} \tag{1}$$

Then, they assumed that the absorbed energy U_a is dissipated in compressing the core and then obtained the ratio of the absorbed energy to the initial kinetic energy $I^2/(2\rho_f h)$ of the upper face sheet neglecting the rate effect within the core

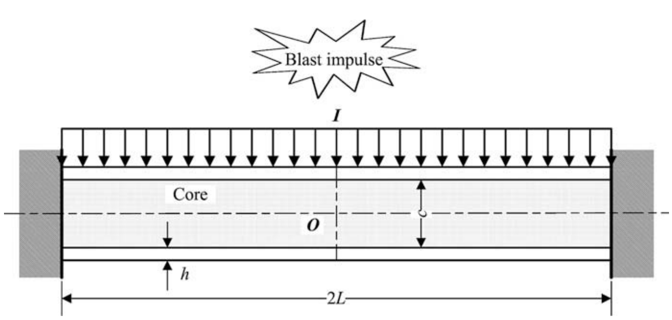

Fig. 1. Sketch of a fully clamped metallic sandwich beam subjected to blast loading.

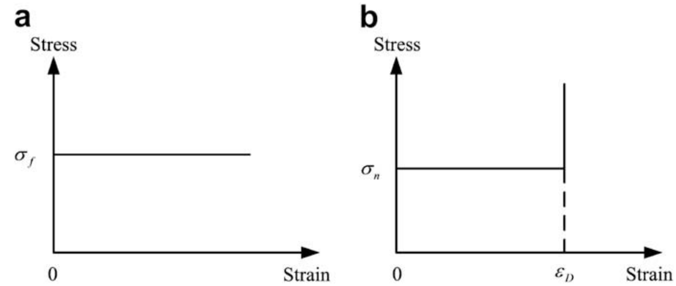

Fig. 2. Stress and strain relations of the face sheets (a) and the metallic core (b) of the metallic sandwich beam.

$$\frac{U_a}{I^2/\left(2\rho_f h\right)} = \frac{1+q}{2+q} \tag{2}$$

where $q = \rho_c c/(\rho_f h)$ is the mass ratio of the core to a face sheet.

Moreover, neglecting the rate effect and considering the plastic energy dissipation in compressing the core at a stress σ_n, they obtained the average compressive strain ε_c over the entire thickness of the core,

$$\varepsilon_c = \frac{\bar{I}^2}{2\bar{\sigma}_n \bar{c}^2 \bar{h}} \times \frac{\bar{h}+\bar{\rho}}{2\bar{h}+\bar{\rho}} \tag{3}$$

where $\bar{I} = I/(L\sqrt{\sigma_f \rho_f})$, $\bar{\sigma}_n = \sigma_n/\sigma_f$, $\bar{c} = c/L$, $\bar{h} = h/c$ and $\bar{\rho} = \rho_c/\rho_f$. The height of the core becomes $c' = c(1-\varepsilon_c)$ if the core has the strain ε_c. The critical value of ε_c is ε_D. The above analysis based on the global energy and the momentum conservation law doses not explicitly capture some deformation mechanism in detail, e.g. the progression of the core compression with the time, but it does assume the existence of the deformation mechanisms.

2.1. Governing equations for small deflection

At the end of the stage of core compression, motion of the fully clamped sandwich beam may be separated into two phases, as shown in Fig. 3, which is similar to the solid monolithic beam (Symonds, 1954). In phase I, there is a central portion in the sandwich beam which translates at the common velocity v_f determined by Eq. (1) and the other segments ξ of the sandwich beam rotate about the supports, as shown in Fig. 3(a). Using the symmetry condition of the beam, we consider half of the sandwich beam for simplicity. The axis x is along the length direction and is measured from the end support of the sandwich beam. The plastic bending moment is taken to vary from $-M'_p$ at the support to $+M'_p$ at the end of segments ξ. The plastic bending moment holds a constant in the central flat portion of the beam. Since the shear force is zero at traveling plastic hinges, vertical equilibrium of the stationary outer segment of the sandwich beam dictates that the shear force at the support is zero. Moreover, time increment in curvature occurs only at the ends of the rotating segments. The velocity field for half of the sandwich beam, as shown in Fig. 3(a), is assumed as

$$\dot{w} = \begin{cases} \dfrac{x}{\xi} v_f, & 0 \le x \le \xi \\ v_f, & \xi \le x \le L \end{cases} \tag{4}$$

Considering the moment of momentum for half of the fully clamped sandwich beam with respect to the fixed end support at time t, we have