Available online at www.sciencedirect.com
ScienceDirect
Fluid Dynamics Research 40 (2008) 34–44
FLUID DYNAMICS RESEARCH

A mixer design for the pigtail braid

B.J. Binder[a], S.M. Cox[b,*]

[a] School of Mathematical Sciences, University of Adelaide, Adelaide 5005, Australia
[b] School of Mathematical Sciences, University of Nottingham, Nottingham NG7 2RD, UK

Received 1 June 2006; received in revised form 17 August 2006; accepted 14 September 2006
Available online 18 October 2006
Communicated by A.D. Gilbert

Abstract

The stirring of a body of viscous fluid using multiple stirring rods is known to be particularly effective when the rods trace out a path corresponding to a nontrivial mathematical braid. The *optimal* braid is the so-called "pigtail braid", in which three stirring rods execute the usual "over-under" motion associated with braiding (plaiting) hair. We show how to achieve this optimal braiding motion straightforwardly: one stirring rod is driven in a figure-of-eight motion, while the other two rods are baffles, which rotate episodically about their common centre. We also explore the extent to which the physical baffles may be replaced by flow structures (such as periodic islands).

Keywords: Chaotic advection; Braid; Stokes flow; Fluid mixing

1. Introduction

This paper concerns laminar mixing of a viscous fluid in a batch mixer. A simple conceptual model for a batch mixer comprises a vat of fluid which is stirred by means of a number of stirring rods. The central design problem, addressed here, is then to devise an effective *stirring protocol* for the mixer, i.e., a manner in which the stirring rods should move to maximize some measure of the mixing quality.

* Corresponding author. Tel.: +44 115 951 4930; fax: +44 115 951 3837.
E-mail addresses: ben.binder@adelaide.edu.au (B.J. Binder), stephen.cox@nottingham.ac.uk (S.M. Cox)
URLs: http://www.maths.adelaide.edu.au/applied/staff/bbinder.html (B.J. Binder),
http://www.maths.nottingham.ac.uk/personal/smc/ (S.M. Cox).

0169-5983/$32.00
doi:10.1016/j.fluiddyn.2006.09.001

It is well known (Aref, 1984, 2002) that even laminar fluid flows can stir a fluid effectively, provided that the Lagrangian particle paths are chaotic. However, it is an unfortunate fact of life that the stirring quality of the resulting flow can be drastically altered by changes to the parameters such as the relative diameters of the vat and rods, the precise path followed by the stirring rods while executing their protocol, or the cross-sectional profiles of the vat or rods.

Some renewed hope of designing *robust* mixing devices, which are not so sensitive to the vagaries of "accidental" parameter variations was provided recently by Boyland et al. (2000), who pointed out, on the basis of Thurston–Nielsen theory, that a certain rate of material line stretch can be guaranteed if at least three stirring rods are used, and if they execute a motion corresponding to a nontrivial mathematical braid. The resulting flows appear in practice to possess a significant region in which the lower bound on the line stretch rate is achieved or exceeded (Finn et al., 2003), given only the topology of the boundary motions that generate the flow (although the underlying Thurston–Nielsen theory does not predict the size of this region, merely its existence). A recent review is provided by Thiffeault and Finn (2006).

For three stirring elements, the optimal braid word, in the sense that it maximizes an appropriately defined entropy, is the "pigtail braid" (D'Alessandro et al., 1999). (We stress here that the concept of optimality is used throughout this paper only in a certain restricted sense, which is explained more explicitly below, at the end of Section 2.) However, it seems difficult to design a mixer that achieves such a braid and is also simple to construct. We demonstrate in this paper one straightforward means for accomplishing the pigtail braid using simple technology: a single stirring rod together with a pair of baffles that can rotate on a turntable.

The structure of the paper is as follows. In Section 2 we briefly introduce some concepts and notation associated with mathematical braids, and summarize some results pertinent for our fluid mechanical application. In Section 3 we demonstrate our design for implementation of the pigtail braid. Numerical simulations of Stokes flow in our device are described in Section 4, where we compare our results with theoretical predictions. We shall see that flow structures such as periodic orbits (and associated regular islands in the flow) act as barriers to transport and thus effectively serve as additional baffles; such flow structures have recently been named "ghost rods" (Gouillart et al., 2006). Our conclusions are drawn in Section 5.

2. Mathematical background

We consider batch mixing devices that comprise a vat of fluid stirred by m stirring elements. These stirring elements are either mobile stirring rods or baffles. (It turns out, somewhat counterintuitively, that the judicious introduction of baffles, usually stationary, can significantly improve the mixing.) Our model will assume two-dimensional flow, and to simplify matters still further, the vat and the stirring rods will all have circular cross-section.

As the stirring rods move, the topology of their motion relative to each other and to the baffles is usefully described using the mathematical language of braids. We therefore begin by briefly recalling some necessary terminology relating to braids. A more comprehensive account, again from the perspective of applications in fluid mechanics, is given in Boyland et al. (2000)—see also Mackay (2001).

We suppose that initially the stirring elements are arranged in some order, from left to right according to some observer. Then the motion of the stirring rods may be characterized topologically in terms of successive interchanges in their order. For instance, the interchange of the jth and $(j+1)$th stirring