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A unified equation for creep-fatigue

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ABSTRACT

"Pure fatigue" is a special case of creep-fatigue; and the Coffin–Manson equation, $\Delta\varepsilon_p = C_o N^{-\beta_o}$, is a special case of the general creep-fatigue equation, which is proposed to take the form: $\Delta\varepsilon_p = C_o s(\sigma) c(T,f) N^{-\beta_o b(T,f)}$. The functions, $s(\sigma)$, $c(T,f)$ and $b(T,f)$, embody the stress–time–temperature characteristic of creep. At the reference condition when creep is dormant, $s(\sigma) = c(T,f) = b(T,f) = 1$, the Coffin–Manson equation is recovered. At the extreme condition when $c(T,f) = 0$, creep-rupture occurs without fatigue. In between these two extreme conditions whence $0 \leqslant c(T,f) \leqslant 1$, creep-fatigue prevails.

1. Introduction

In advanced microelectronic assembly such as ball grid array packaging, the integrated circuit (IC) component, the solder joints, and the printed circuit board (PCB) form a three-layer construction. As the electronic device is powered on and off, the IC chip within the IC component experiences heating and cooling leading to thermal expansion and contraction of the IC component. The mismatch in thermal expansion between the IC component and the PCB is accommodated by the interconnecting solder joints resulting in creep-fatigue.

The creep-fatigue of solder materials and solder joints in the microelectronic assembly have been extensively studied over the last three decades and a number of life prediction models have been proposed [1–4]. These models are invariably traced back to prior studies in the power and the transport industries that span fifteen decades. The systematic investigation of fatigue leading to the stress-life (S–N) curve for high cycle fatigue can be dated to Wöhler [5]. Basquin [6] described the stress-life data in the form of a power-law. The extension of stress-life to strain-life for low-cycle fatigue was reported by Coffin [7], and independently by Manson [8]. This led to the Coffin–Masson equation

$$\Delta\varepsilon_p = C_o N^{-\beta_o} \tag{1}$$

where C_o and β_o are constants, referred to as the fatigue ductility coefficient and the fatigue ductility exponent, respectively. The studies of creep-fatigue began in 1950s pioneered by the power and the aviation industries [9]. It seems that the Coffin–Masson equation can be extended to creep-fatigue by simply replacing the plastic strain range $\Delta\varepsilon_p$ in Eq. (1) with the inelastic strain range $\Delta\varepsilon_{in}$, which is the sum of the plastic strain range $\Delta\varepsilon_p$ and the creep strain range $\Delta\varepsilon_c$; that is,

$$\Delta\varepsilon_p \rightarrow \Delta\varepsilon_{in} = \Delta\varepsilon_p + \Delta\varepsilon_c \tag{2}$$

However, there are two complications:

<u>*Non- additivity of inelastic strains*</u>

The physics of damage are very different for cyclic fatigue and creep [10–12]. Cyclic strain induces damages through formation of slips in the lattice of the material leading to the formation of persistent slip bands as exemplified by the formation of intrusions and extrusions on the surface of the structure; alternations of these intrusions and extrusions lead to the nucleation of microcracks. In contrast, creep induces damage through diffusion of dislocations along the grain boundaries ($0.4\ T_m < T < 0.6\ T_m$) and within the lattice ($T > 0.6\ T_m$) that accumulate into micro-voids, which do not necessarily occur on the surface of the structure. Microstructurally, fatigue cracking is typically trans-granular with damaged confined to the slip planes while creep damage is inter-granular and dispersed. The different nature of damages suggests that the plastic strain range $\Delta\varepsilon_p$ and creep strain range $\Delta\varepsilon_c$ are not additive as suggested by Eq. (2).

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Indefiniteness of creep strain

The general expression of creep strain rate takes the form

$$\dot{\varepsilon}_c = f(M, \sigma, T, t, \varepsilon) \quad (3)$$

or assuming the various effects are separable,

$$\dot{\varepsilon}_c = f_1(M) f_2(\sigma) f_3(T) g_1(t) g_2(\varepsilon) \quad (4)$$

where $f_1(M)$ represents the microstructure of the test specimen, which might take the form [13]

$$f_1 = \frac{Gb}{kT}\left(\frac{b}{d}\right)^p \quad (5)$$

where, G, d, and b are shear modulus, grain size and Burgers vector, respectively, and p is a fitting constant. In practice, however, the role of microstructure and its evolution during the process of creep are typically ignored due to the difficulty of its characterisation in a test specimen and its implementation in an analysis. The functions $g_1(t)$ and $g_2(\varepsilon)$ embody the hypotheses of time hardening and strain hardening, respectively, during the primary phase of creep. In practice, neither time hardening nor strain hardening is satisfactory, and a reliable hardening model for creep strain remains elusive [12]. Researchers in the microelectronic assembly community typically assume the solder joints exhibit only steady-state creep, which implies $g_1(t) = g_2(\varepsilon) = 1$, thus circumventing (but not overcoming) the challenge. The accuracy of the evaluated creep strain hence must suffer from these gross simplifications.

A number of alternative life prediction methods have been proposed. These include [12]:

- The hysteresis energy method [14], which suggests that the inelastic damage accumulated in a cycle is given by the tensile portion of the stress–strain hysteresis energy:

$$w = \int_{tension\,cycle} \sigma \varepsilon_{in} d\varepsilon \quad (6)$$

where w may be used in place of $\Delta\varepsilon_{in}$ in Eq. (2). Inherent in this method is the assumption of the linear additivity of the plastic and the creep strains. A similar method that uses the entire stress–strain hysteresis energy, not just the tension portion, is often used in the microelectronic assembly community for modelling the creep-fatigue of solder joints [15].

- The linear damage summation rule [16], which assumes that damage, regardless of whether it comes from creep or fatigue, is cumulative in a linear fashion. That is, failure occurs when

$$D_f + D_c = 1 \quad (7)$$

where, $D_f = \sum_i \frac{n_i}{N_i}$ is the damage caused by fatigue [17], $D_c = \sum_i \frac{t_i}{t_{Ri}}$ is the damage due to creep based on the life-fraction rule [18] or $D_c = \sum_i \frac{\varepsilon_{ci}}{\varepsilon_{Ri}}$ based on strain-fraction [19]; where n_i is the number of cycles at a given strain range $\Delta\varepsilon_p$; N_i is the fatigue life at the same strain range at a temperature at which the mechanism of creep is inactive; t_i and ε_{ci} are the time and the creep strain, respectively, at a given stress and temperature; t_R and ε_R are the time-to-rupture and rupture strain, respectively, at the same stress and temperature. The assumed linear addition of damage is inconsistent with the microstructural characteristics of fatigue damage and creep damage [10–12,20]. Despite its inaccuracy [12,21], this method is popularly used owing to its simplicity. However, this method is not used in the microelectronic assembly community for the simple reason that it is impractical to characterise the strain-life of microelectronic assembly materials at the temperature at which the mechanism of creep is dormant; e.g., the melting temperature, T_m, of the eutectic SnPb solder is 456 K and 0.4 T_m is 110 °C below room temperature.

- The strain range partitioning method [22], which apportions the damage within a cycle using the following rule:

$$\frac{1}{N} = \frac{\Delta\varepsilon_{pp}}{\Delta\varepsilon_{in}N_{pp}} + \frac{\Delta\varepsilon_{cc}}{\Delta\varepsilon_{in}N_{cc}} + \frac{\Delta\varepsilon_{cp}}{\Delta\varepsilon_{in}N_{cp}} + \frac{\Delta\varepsilon_{pc}}{\Delta\varepsilon_{in}N_{pc}} \quad (8)$$

where $\Delta\varepsilon_{pp}$ is plastic strain reversed by plastic strain, $\Delta\varepsilon_{cc}$ is tensile creep strain reversed by compressive creep, $\Delta\varepsilon_{cp}$ is tensile creep reversed by compressive plasticity, and $\Delta\varepsilon_{pc}$ is tensile plasticity reversed by compressive creep. Individual strain components are assumed to obey their respective "Coffin–Manson" relation; i.e., $N^{\beta_{jj}} = C_{jj}^{-1}\Delta\varepsilon_{jj}$. An analogous energy partitioning method has been investigated for modelling the creep-fatigue of solder joints in microelectronic assembly [23].

- The fracture mechanics based method [24], which rests on the observation that low-cycle fatigue is dominated by crack propagation and suggested that the rate of crack growth is driven collectively by cyclic fatigue and creep; that is,

$$\left.\frac{da}{dN}\right|_{cycle} = \left.\frac{da}{dN}\right|_{fatigue} + \left.\frac{da}{dt}\right|_{cycle} \quad (9)$$

where $\left.\frac{da}{dN}\right|_{fatigue} = C_1 \Delta J_{eff}^{n/2}$ and $\left.\frac{da}{dt}\right|_{cycle} = \int_{tensioncycle} C_2 C^{*n'} dt$; ΔJ_{eff} is the effective range of J-integral; and C^* is the time-dependent fracture parameter. A method frequently used for modelling solder joints assumes crack initiation and propagation are power-law functions of the total stress–strain hysteresis energy [25].

- The mechanism-based method [26], which assumes that fatigue damage is characterised by crack size, a, and creep damage by cavity size c, each governed by respective equations:

$$\frac{1}{a}\frac{da}{dt} = \begin{Bmatrix} T \\ C \end{Bmatrix}\left(1 + \alpha \ln\frac{c}{c_o}\right)|\varepsilon_{in}^m||\dot{\varepsilon}_{in}^k|$$

$$\frac{1}{c}\frac{dc}{dt} = \begin{Bmatrix} G_T \\ -G_C \end{Bmatrix}|\varepsilon_{in}^m||\dot{\varepsilon}_{in}^k|$$

where T and G_T are for tension, C and $-G_C$ for compression, and c_o is a threshold cavity size below which cavities will be sintered away.

- The frequency modified method [27], which accounts for the effects of creep through the introduction of a frequency term into the Coffin–Manson equation:

$$\Delta\varepsilon_p = C_c(Nf^{k-1})^{-\beta_o} \quad (11)$$

where C_c, k, and β_o are constants. Comparing Eq. (11) with Eq. (1) suggests that the fatigue ductility coefficient C_o is now a function of frequency given by $C_c f^{\beta_o(1-k)}$.

The simplicity of the frequency modified method has attracted significant interests among the microelectronic assembly community [28–31]. Solomon [29] and Kanchanomai et al. [31] have studied the individual effect of temperature and frequency on the fatigue life of solder, while Engelmaier [28] and Shi et al. [30] have integrated temperature and frequency into the Coffin–Manson equation for solder.

The first study in the frequency sensitivity of the fatigue life of solder was reported by Solomon [29], who performed the lap shear test on eutectic SnPb solder at 35 °C and reported the values of the frequency exponent in Eq. (11) to be $k = -0.42$ for the frequency range 5×10^{-5} Hz < f < 3×10^{-4} Hz and $k = 0.84$ for the frequency range 3×10^{-4} Hz < f < 3×10^{-1} Hz. Shi et al. [30] tested the eutectic SnPb solder under uniaxial tension-compression loading at 25 °C at frequencies from 10^{-4} Hz to 1 Hz, and reported $k = 0.42$ for 10^{-4} Hz < f < 10^{-3} Hz and $k = 0.91$ for 10^{-3} Hz < f < 1 Hz. The reported discontinuity of the frequency exponent k by Solomon [29] and Shi et al. [30] is unintuitive and remained unexplained.

From another series of tests at 1 Hz and at five temperatures between −40 °C and 150 °C, Shi et al. [30] then expressed the