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A small-load-omitting criterion based on probability fatigue

C.J. Wang [a,*], W.X. Yao [b], T.X. Xia [a]

[a] Key Laboratory of Fundamental Science for National Defense-Advanced Design Technology of Flight Vehicle, Nanjing University of Aeronautics & Astronautics, Nanjing 20016, China
[b] State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

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ABSTRACT

A small-load-omitting criterion (SLOC) which is evaluated by whether the truncated spectrum has the same fatigue life probability distribution as the original spectrum is developed. Primary focus is placed on the uncertainties of fatigue properties and fatigue life. Probabilistic distance is used to measure the difference between two fatigue life probability distributions. The omission level can be obtained when the probabilistic distance is equal to a tolerance value predefined. Fatigue tests with smooth sheet specimens made of LC4CS Aluminum alloy are conducted to verify this criterion. It is found that experimental results agree well with theoretical solutions.

1. Introduction

The load-time histories of structure often contain a large percentage of small amplitude cycles [1–4]. Removing these small cycles, which are considered to be non-damaging, is of great importance for saving testing time and cost. However, there is no final conclusion on how to remove these small amplitude cycles. Based on the test results of several load spectra, Heuler and Seeger regarded 50% of the endurance limit (10^7 cycles) as the omission level [5]. de Jonge and Nederveen tested 2024-T3 sheet specimens under TWIST and Mini-TWIST load sequences, showing that omission of the lowest gust load cycles (81% of the fatigue limit) resulted in a crack initiation life increase by a factor of 2.4 [6]. Based on the test results of 45 steel notched elements under three load spectra, Yan et al. presented a small-load-omitting criterion (SLOC), indicating that $(\Delta\sigma_{eqv})_{\text{th}}$, the FCI (fatigue crack initiation) threshold, could be taken as the omission level [7]. The influence of different low load truncation level (9.82%, 11.72%, 13.98%, 17.11%, and 21.36% of the biggest load cycle) on crack growth for Al 2324-T39 and Al 7050-T7451 was studied by means of comparative fatigue tests of middle-crack tension specimens under six different flight-by-flight spectra [8]. It was shown that an omission level of 11.72% or 13.98% was reasonable. The test results of LY12CZ and 30CrMnSiNi2A sheet specimens under fighter spectra demonstrated that omission of low amplitude cycles with an overload ratio ($\sigma_{max,max}/\sigma_{max}$) $r \geqslant 2.5$ had no distinct influence on crack initiation life [9].

In the existing SLOCs which are based on deterministic fatigue-damage evaluation, load cycles are identified into “damaging” and “non-damaging” cycles by an omission level (fatigue limit or other stress amplitude). However, due to the uncertainty of fatigue properties, damage induced by a stress cycle is a probabilistic value rather than “damaging” or “non-damaging”. It is deficient for these SLOCs to reveal the essence. In the theory of probability fatigue, fatigue life is a random variable. A reasonable SLOC should ensure that there is no significant difference between the fatigue life probability distributions of truncated and original spectra. In order to solve this problem theoretically, two issues should be addressed. The first one is how to describe the difference between two probability distributions, and the other one is how to calculate the fatigue life and its probability distribution under a specified spectrum. Taking the randomness of fatigue properties and fatigue life into account, we develop a SLOC which is evaluated by whether the truncated spectrum has the same fatigue life probability distribution as the original spectrum.

2. Generation of small-load-omitting criterion

2.1. Small-load-omitting principle

The evaluation criterion of the existing SLOCs is whether the truncated spectrum has the same mean value of fatigue life as the original one. For a given omission level S_{omit}, it is considered to be reasonable if there is little difference between the mean

* Corresponding author. Tel./fax: +86 25 84892177.
E-mail address: wcj007306@aliyun.com (C.J. Wang).

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values of fatigue life of the truncated and the original spectra. With increasing requirements of structure reliability, it is not enough to just consider the mean value in fatigue design. Here a SLOC based on probability fatigue is developed. For any specified spectrum and a given S_{omit}, eliminating small cycles of which amplitudes are less than S_{omit}, it is considered to be reasonable if the fatigue life probability distribution of the truncated spectrum is the same as that of the original spectrum.

Let F be the fatigue life probability distribution of original spectrum, and G be the corresponding fatigue life probability distribution by eliminating small cycles of which amplitudes are less than S_{omit}. If for F and G.

$$D(F||G) \leqslant \delta \tag{1}$$

it is considered that F and G have the same probability distribution, and eliminating the small cycles has no significant influence on fatigue life. Where $D(F\|G)$ is probabilistic distance used to measure the difference between two probability distributions F and G, and δ is tolerance value.

2.2. Measure of the difference between two fatigue life probability distributions

Probabilistic distance, which is used to measure the similarity of two probability distributions, can be represented in different forms, such as Kullback–Leibler divergence (KL divergence), Kolmogorov metric, total variation metric and Hellinger distance [10]. Among these probabilistic distances, the KL divergence is very popular in statistics. For distributions F and G of a continuous random variable, KL divergence is defined to be the integral as follows

$$D_{KL}(F||G) = \int f(x) \ln \frac{f(x)}{g(x)} dx \tag{2}$$

where f and g denote the densities of F and G respectively. The KL divergence is always non-negative, and is equal to zero if and only if $F = G$. Obviously, the KL divergence can be used to measure the difference between the fatigue life probability distributions of the truncated spectrum and the original spectrum.

It is generally accepted that fatigue life follows a log-normal or Weibull distribution. The KL divergence in Eq. (2) can be integrated [11]

$$\begin{aligned} D_{KL}(F||G) &= \frac{1}{2}\left[\ln\frac{\gamma^2}{\sigma^2} + \frac{\sigma^2}{\gamma^2} - 1 + \frac{(\mu-v)^2}{\gamma^2}\right] \quad \text{(lognormal)} \\ D_{KL}(F||G) &= \ln\left(\frac{\alpha_1}{\alpha_2}\cdot\frac{\beta_2^{\frac{\alpha_2}{\alpha_1}}}{\beta_1^{\frac{\alpha_2}{\alpha_1}}}\right) + r\left(\frac{\alpha_2-\alpha_1}{\alpha_1}\right) + \frac{\beta_1^{\frac{\alpha_2}{\alpha_1}}}{\beta_2}\Gamma\left(1+\frac{\alpha_2}{\alpha_1}\right) - 1 \quad \text{(Weibull)} \end{aligned} \tag{3}$$

If F and G are lognormal distributed, μ and σ are the logarithmic mean value and standard deviation of the distribution F, v and γ are the logarithmic mean value and standard deviation of the distribution G. If F and G are Weibull distributed, α_1 and β_1 are the shape parameter and scale parameter of the distribution F, α_2 and β_2 are the shape parameter and scale parameter of the distribution G. These distribution parameters can be solved by a fatigue reliability assessment method. Although the fatigue life distribution type makes the KL divergence form changed, the remainder procedure of the developed SLOC is the same whether log-normal or Weibull distribution is used. For simplicity, only lognormal distribution is used to illustrate the SLOC in the following part.

2.3. Calculation of fatigue life distribution – variable amplitude loading

The factors that influence the fatigue life variation can be classified mainly into inherent and external scatter characteristic [12]. The inherent scatter characteristics refer to the material performance variability that is caused by such factors as inhomogeneities of material microstructure, composition substance, and defect distribution, etc., which can only be obtained from fatigue tests. The external scatter characteristic refers to the variability of structure geometry, operational condition, and fatigue load, etc. Due to the variability of structure geometry, even under the same load, the stress of the critical point may be a random variable, which causes the difference between the fatigue performance of material and that of structure. The variability of fatigue load is reflected in two aspects, the randomness of load sequence and the distribution of load cycles. Mechanical component always operates in a cyclic or approximate cyclic condition. The distribution of load cycles can be obtained by the rainflow counting method from a recorded load history, and this distribution would be stable if the record time is long enough. From this point, the distribution of load cycles is actually a determined probability distribution, which has no randomness. Therefore, the main role of the variability of fatigue load is the randomness of load sequence. In a word, the factors that influence the fatigue life variation mainly include the variability of material performance, structure geometry and load sequence.

The material fatigue performance is always defined by probabilistic stress-life (p–S–N) curves. Considering the variability of structure geometry, the p–S–N curves of a component can be obtained on the basis of the p–S–N curves of material [13,14]. For a specified spectrum, the distribution of load cycles is given, and there is only the randomness of load sequence. Fatigue reliability analysis of the component can be solved by combining a random fatigue accumulative damage rule and the p–S–N curves of the component. According to the two-dimensional probabilistic (TP) Miner's rule [15], the fatigue life under certain reliability p is

$$T_p \cdot \sum_i \sum_j \frac{n_{ij}}{N_{cp}(S_{ai}, S_{mj})} = 1, \quad N_{vp} = T_p \cdot \sum_i \sum_j n_{ij} \tag{4}$$

$$N_{vp} \cdot \iint_Q \frac{l(S_a, S_m)}{N_{cp}(S_a, S_m)} dS_a dS_m = 1 \tag{5}$$

where n_{ij} is the cycle number of (S_{ai}, S_{mj}) in variable amplitude loading, S_a is the amplitude stress of load cycle, S_m is the mean stress of load cycle, T_p is the variable amplitude fatigue life in 'blocks to failure', N_{cp} is the constant amplitude fatigue life, N_{vp} is the variable amplitude fatigue life in 'cycles to failure', T_p, N_{cp} and N_{vp} correspond to the reliability p, $l(S_a, S_m)$ is the distribution density function of (S_a, S_m). On the basis of TP Miner's rule, fatigue reliability analysis can be carried out only with p–S–N curves and load spectrum. However, the load sequence effect is worth further studied. In [16], it is shown that the load sequence has no significant influence on the fatigue life for a typical commercial fixed-wing fatigue load spectra. Hence, the load sequence effect is not taken into account in this study, and the TP Miner's rule is used to calculate the distribution parameters of fatigue life in variable amplitude loading.

Let $l(S)$ be the distribution density function of stress amplitude. According to the TP Miner's rule, fatigue lives of the original spectrum and the truncated spectrum under the reliability p are

$$\begin{aligned} N_{vp\_ori} &= \frac{1}{\int_0^\infty \frac{l(S)}{N_{cp}(S)} dS} \\ N_{vp\_tra} &= \frac{1}{\int_{S_{omit}}^\infty \frac{l(S)}{N_{cp}(S)} dS} \end{aligned} \tag{6}$$

where N_{vp_ori} and N_{vp_tra} are the fatigue lives of the original spectrum and the truncated spectrum respectively. $N_{cp}(S)$ can be obtained by comparing the stress S with the S–N curve. A three-parameter expression for p–S–N curves is assumed as

$$N_{cp}(S - S_{0p})^{H_p} = C_p \tag{7}$$

where S_{0p} is the theoretical fatigue limit, H_p and C_p are material constants.