European Journal of Mechanics A/Solids 28 (2009) 569–581

Contents lists available at ScienceDirect

European Journal of Mechanics A/Solids

www.elsevier.com/locate/ejmsol

# Propagation characteristics of thermoelastic waves in piezoelectric (6 mm class) rotating plate

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ARTICLE INFO

*Article history:*
Received 20 March 2008
Accepted 3 September 2008
Available online 10 September 2008

*Keywords:*
Angular velocity
Rayleigh and Lamb waves
Thermal relaxation
Iteration method
Sensors

ABSTRACT

The propagation of Lamb waves in a homogeneous, transversely isotropic (6 mm class), piezothermoelastic plate rotating with uniform angular velocity about normal to its boundary has been investigated. The generalized (non-classical) theories of thermoelasticity in contrast to Sharma and Pal [Sharma, J.N., Pal, M., 2004. Lamb wave propagation in transversely isotropic piezothermoelastic plate. J. Sound Vib. 270, 587–610] have been used to investigate the problem. The surfaces of the plate are subjected to stress free, thermally insulated/isothermal and electrically shorted boundary conditions. Secular equations for wave propagation modes in the plate are derived from a coupled system of governing partial differential equations of linear piezothermoelasticity. After obtaining the complex characteristic roots with the help of Descartes' algorithm, the transcendental secular equations have been solved by functional iteration numerical technique to compute phase velocity and attenuation coefficient. Finally, in order to illustrate the analytical development, numerical solution of secular equations is carried out for PZT-5A piezothermoelastic material. The corresponding simulated results of various physical quantities such as phase velocity, attenuation coefficients, specific loss factor of energy dissipation, thermo-mechanical coupling factor and relative frequency shifts have been presented graphically for both rotating and non-rotating plates for comparison purpose. There is a scope for extension of the present work to other classes of piezo/pyroelectric crystals. The study will be useful in design and construction of gyroscope, rotation sensors, temperature sensors and other pyro/piezoelectric surface acoustic wave (SAW) devices.



## 1. Introduction

The piezoelectricity was discovered by the brothers Curie in 1880 (Curie and Curie, 1880). Propagation of waves in piezoelectric plates has been an active research area for several decades because of the application in piezoelectric transducers, resonators, filters, actuators and other devices such as microelectromechanical systems (MEMS). A number of exact solutions of the three-dimensional dynamical equations have been obtained for widely used materials such as ceramics, various crystal cuts of quartz and materials of other symmetries. The detailed studies and analysis of piezoelectric vibratory gyroscopes can be found in the recent publications by Yang et al. (1998). A comprehensive review of the work on piezoelectricity and related fields has been done by Yang et al. (2003, 2005).

The thermo-piezoelectricity theory was first proposed by Mindlin (1979) who derived the governing equations for a thermo-piezoelectric plate. The effect of rotation on elastic waves, both partial and surface, has been studied by many authors, such as Clarke and Burdess (1994), Wren and Burdess (1987) and Soderkvist (1994). Ting (2004) investigated the interfacial waves in a rotating anisotropic elastic half space by extending the Stroh (1962) formalism. He obtained explicit expressions of the polarization vector and the secular equation for monoclinic material halfspace rotating about the normal to the plane of symmetry. Fang et al. (2002) investigated the effect of rotation on surface acoustic waves in a piezoelectric halfspace. It is shown that a piezoelectric material may not permit propagation of more than one rotation-perturbed surface wave even if both Rayleigh and Bleustein–Gulyaev waves are permissible in the absence of rotation. Fang et al. (2000) studied the effect of rotation on the characteristics of waves propagating in a piezoelectric plate. The dispersion relation for polarized ceramic plates was analyzed in details in view of the gyroscope applications. Zhou and Jiang (2001) studied the effects of Coriolis and centrifugal forces on acoustic waves in a piezoelectric half-space. Yang et al. (1998) studied thickness vibrations in a rotating piezoelectric plate. Sharma and Thakur (2006) studied the effect of rotation on Rayleigh–Lamb waves in magneto-thermoelastic plates. Sharma and Othman (2007) investigated the effect of rotation on generalized thermo-viscoelastic Rayleigh–Lamb waves in plates. Sharma and Kumar (2000) have studied the propagation of plane harmonic waves in

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0997-7538/$ – see front matter 
doi:10.1016/j.euromechsol.2008.09.001

**Nomenclature**

| | |
|---|---|
| $\rho$ | density |
| $C_e$ | specific heat at constant strain |
| $\epsilon$ | thermoelastic coupling constant |
| $e_{ij}$ | piezoelectric constants |
| $\epsilon_{ij}$ | electric permittivity |
| $\delta_{jk}$ | Kronecker's delta |
| $h$ | surface heat transfer coefficient |
| $\vec{u}(r,z,t) = [u, 0, w]$ | displacement vector |
| $T$ | temperature change |
| $T_0$ | initial temperature |
| $C_e$ | specific heat at constant strain |
| $c_{ij}$ | are isothermal elastic tensor |
| $\xi$ | the wave number |
| $\Delta\omega = \omega(\Omega) - \omega(0)$ | frequency shift |
| $D$ | electrical displacement |
| $\phi$ | electric potential |
| $\sigma_{ij}$ | stress tensor |
| $W$ | elastic energy |
| $E_i$ | the electric field |
| $\Omega_i$ | the uniform angular velocity of rotation |
| $\kappa^2$ | the electromechanical coupling factor |
| $v$ | phase velocity of wave |
| PZT-5 | Lead–Zirconate–Titanate |
| $\beta_1, \beta_3$ | are the isothermal thermoelastic parameter |

piezo-thermoelastic materials. Sharma and Pal (2004) investigated the propagation of Lamb waves in a transversely isotropic, charge- and stress-free piezothermoelastic plate in the context of conventional coupled theory of piezothermoelasticity. They studied the wave characteristics, such as phase velocity and attenuation coefficient of the waves in Cadmium Selenide (CdSe) material. Recently, Sharma and Walia (2008) have studied the effect of rotation on Rayleigh waves in piezothermoelastic half space.

According to Yang and Fang (2002, 2003) the piezoelectric gyroscopes are very small than devices with sizes in the order of 1–2 cm and resonant frequency of 10 kHz–100 kHz. The angular velocity to be measured is quite smaller than the resonant frequencies of the working modes of the gyroscopes. In this situation the Coriolis force being linear is responsible for the sensing mechanism of the gyroscopes instead of the centrifugal force which is very small being quadratic in angular frequency and hence can be neglected. However, centrifugal force plays a vital role on the vibrations of a rotating elastic body in machine dynamics where large bodies with low resonance frequencies are in relatively fast rotations with the centrifugal force as one of the dominant forces. Although the effect of thermal relaxation times on phase velocity, attenuation coefficient and specific loss factor of energy dissipation on Lamb waves in piezothermoelastic plates have been studied by Sharma and Walia (2006) in order to furtherance the work of Sharma and Pal (2004), yet there is a need to investigate the effect of rotation on various characteristics of Lamb waves in the considered materials under different conditions from the technical point of view. Some additional characteristics such as thermomechanical coupling factor and relative frequency shift due to rotation are also needed to be explored. It is also mentioned here that the plane waves (incident rays) are reflected within the inner faces of the piezoelectric plate (dielectric sheet) leading to the generation of Lamb waves in the plate. Owing to the high thickness value as compared with the wavelength and plane wave spot size, the paths of the different reflected waves are assumed to be sufficiently far apart so that interference does not occur.

Keeping these facts in view, an attempt has been made to investigate the propagation of Lamb waves in a homogeneous, transversely isotropic, piezothermoelastic (6 mm class) plates in context of non-classical (generalized) theories of thermoelasticity. The plate is assumed to be rotating with uniform angular velocity about its normal. The surfaces of the plate are subjected to stress free, thermally insulated/isothermal and closed circuit boundary conditions. A hybrid algorithm consisting of direct method (Descartes Method) for solving fourth degree complex polynomial equation has been used to compute the characteristics roots involved in the formal solution. These roots are utilized to solve secular equations by using functional iterative numerical technique to obtain various wave characteristics of the considered waves. Finally, in order to illustrate the analytical development, numerical solution of secular equations is carried out for PZT-5A piezothermoelastic material in contrast to CdSe in earlier works. The computer simulated results in respect of phase velocity, attenuation coefficient and specific loss factor of energy dissipation, thermomechanical coupling factor and relative frequency shift have been presented graphically.

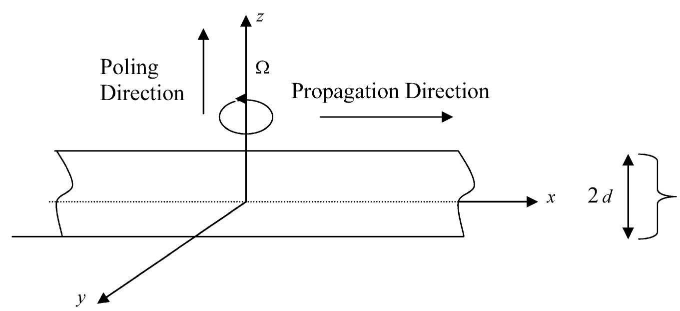


**Fig. 1.** Piezothermoelastic plate rotating about its normal.

## 2. Formulation of the problem

We consider homogeneous, transversely isotropic (6 mm class), generalized piezothermoelastic plate of thickness $2d$ at uniform temperature $T_0$ in undisturbed state. We take origin of the coordinate system $(x, y, z)$ on the middle surface of the plate. The $xy$-plane is chosen to coincide with middle surface and $z$-axis normal to it along the thickness. We choose $x$-axis in the direction of wave propagation so that all the particles on a line parallel to $y$-axis are equally displaced, therefore all the field quantities are independent of $y$-coordinate. The surfaces of plate are given by $z = \pm d$ which are subjected to stress free, thermally insulated/isothermal and electrically shorted (closed circuit) boundary conditions. The plate is rotating about $z$-axis with uniform angular velocity $\vec{\Omega} = [0, 0, \Omega]$. Let $\vec{u}(x,z,t) = [u, 0, w]$ be the displacement vector, $\phi(x,z,t)$ is the electric potential and $T(x,z,t)$ denotes temperature change in the plate. The geometry of the problem is shown in Fig. 1.

Upon including the effect of Coriolis and centrifugal forces due to rotation the non-dimensional basic governing equations for the considered plate, in the absence of charge density, heat sources and body forces in non-dimensional form linear generalized theories of piezothermoelasticity are given as Sharma et al. (2008)

$$u_{,xx} + c_2 u_{,zz} + c_3 w_{,xz} + e_1 \phi_{,xz} - (T + t_1 \delta_{2k} \dot{T})_{,x} = \ddot{u} - \Omega^2 u + 2\Omega \dot{w}, \tag{1}$$

$$c_3 u_{,xz} + c_2 w_{,xx} + c_1 w_{,zz} + (e_2 \phi_{,xx} + \phi_{,zz}) - \bar{\beta}(T + t_1 \delta_{2k} \dot{T})_{,z}$$