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# Notched fatigue behavior and stress analysis under multiaxial states of stress

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ABSTRACT

The effect of notches on multiaxial fatigue behavior was studied using thin-walled tubular 2024-T3 aluminum specimens with a circular transverse hole. Constant amplitude fully reversed axial, torsion, and in-phase and 90° out-of-phase axial–torsion tests were performed in load control. Stress analysis was performed using both Neuber's rule and FEA to study local stress distributions. Neuber's rule was found to be in reasonable agreement with FEA results for all loading conditions considered. Fatigue crack initiation for all loading cases examined was experimentally observed to occur on planes of maximum shear. For the same equivalent nominal stress, experimental lives in pure torsion were longer than those for pure axial loading. In-phase fatigue lives were also longer than out-of-phase lives at the same equivalent stress. The nominal stress–life approach, applied by considering the fatigue notch factor along with both maximum principal stress and von Mises equivalent (or effective) stress, produced mostly non-conservative fatigue life predictions that varied by as much as a factor of 10 from experimental results. The local stress and strain–life approaches, based on von Mises effective stress or strain, resulted in reasonable correlation of test data at shorter fatigue lives, but became increasingly less accurate in the mid to high cycle fatigue regime, where predictions were in error by more than a factor of 10. The Fatemi–Socie critical plane parameter correlated all test data reasonably well (typically within a factor of 3) and is consistent with the experimentally observed damage mechanism for the tested material.



## 1. Introduction

Due to the abundance of engineering components with stress concentrations being subjected to complex multiaxial loading histories, studying multiaxial fatigue of notched members is of great practical importance. In addition, due to the relatively high strength to weight ratio of aluminum, an increasing number of components in key industries, such as automotive and aerospace, are being manufactured from various aluminum alloys. Therefore, it is of special interest to be able to understand the multiaxial fatigue behavior of these materials. Despite the significance of understanding multiaxial notched fatigue behavior, limited literature exists on the subject due to the synergistic complexities involved in studying this topic. A general overview of multiaxial fatigue, including elaboration on key aspects as well as some approximation models for life estimation, can be found in [1,2].

Tipton and Nelson [3] assessed various multiaxial fatigue life prediction models for notched components developed up until the mid-1990s. Among these are equivalent stress and strain approaches, plastic work and energy approaches, and critical plane approaches. A number of notch root stress and strain estimation methods, including generalized forms of Neuber's rule, energy relations, plasticity model routines, and pseudo stress and strain-based approaches, were also discussed. The models were evaluated in the context of constant amplitude proportional and non-proportional loadings. They concluded that traditional multiaxial damage criteria based on effective stresses or strains, such as von Mises, often result in overly non-conservative life predictions for non-proportional loadings. Additionally, they reported that energy-based or stress and strain-based critical plane approaches show much promise in overcoming the shortcomings of the more traditional damage criteria. The notch root stress–strain approaches reviewed provided reasonably good estimates in the case of blunt notches subjected to proportional loadings, but more work was needed in the area of non-proportional loadings. Finally, they emphasized the importance of considering subsurface stress gradient effects in cases where notched components are subjected to multiaxial loading.

A collection of works focused on the analysis of the Society of Automotive Engineers (SAE) biaxial fatigue testing program can be found in [4]. Tipton and Nelson [5] studied the applicability of stress-based approaches to multiaxial fatigue and concluded that their usefulness is limited due to their dependence on fatigue

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**Nomenclature**

| | |
|---|---|
| $a$ | crack length at the notch root |
| $A$ | S–N curve coefficient |
| $b$ | axial fatigue strength exponent |
| $b_o$ | shear fatigue strength exponent |
| $B$ | S–N curve exponent |
| $B'$ | modified S–N curve exponent |
| $c$ | axial fatigue ductility exponent |
| $c_o$ | shear fatigue ductility exponent |
| $E$ | modulus of elasticity |
| $e^*$ | modified nominal strain |
| $F$ | shape factor |
| $f$ | correction coefficient |
| $G$ | shear modulus |
| $k$ | material constant in the FS parameter |
| $K'$ | cyclic axial strength coefficient |
| $K'_o$ | cyclic shear strength coefficient |
| $K_f$ | fatigue notch factor |
| $K_{fq}$ | equivalent fatigue notch factor |
| $K_t$ | elastic stress concentration factor |
| $K_{tq}$ | equivalent stress concentration factor |
| $\Delta K_{th}$ | threshold stress intensity factor range |
| $K_\varepsilon$ | inelastic strain concentration factor |
| $K_\sigma$ | inelastic stress concentration factor |
| $n'$ | cyclic axial strain hardening exponent |
| $n'_o$ | cyclic shear strain hardening exponent |
| $N_f$ | cycles to failure |
| $r$ | hole radius |
| $R_m$ | midsection radius |
| $S$ | equivalent nominal stress |
| $S_A$ | nominal axial stress |
| $S_{NF}$ | stress amplitude at given fatigue life |
| $S_T$ | nominal shear stress |
| $S^*$ | modified nominal stress |
| $t$ | wall thickness |
| $T$ | torque |
| $\gamma$ | shear strain |
| $\gamma'_f$ | shear fatigue ductility coefficient |
| $\gamma_{max}$ | maximum shear strain |
| $\varepsilon$ | normal strain |
| $\varepsilon_q$ | equivalent strain |
| $\varepsilon'_f$ | axial fatigue ductility coefficient |
| $\lambda$ | nominal stress ratio ($S_T/S_A$) |
| $\theta$ | angular location on hole perimeter |
| $\rho$ | material characteristic length |
| $\sigma$ | normal stress |
| $\Delta\sigma_e$ | fatigue limit range |
| $\sigma_{n,max}$ | maximum normal stress on the maximum shear strain plane |
| $\sigma_y$ | yield strength |
| $\sigma_q$ | equivalent stress |
| $\sigma'_f$ | axial fatigue strength coefficient |
| $\tau$ | shear stress |
| $\tau'_f$ | shear fatigue strength coefficient |

notch factor, which is difficult to predict in the absence of experimental data. Similarly, Tipton and Fash [6] evaluated multiaxial fatigue life predictions using eight different strain-based and critical plane approaches. They concluded that for the same applied loads, out-of-phase loading is less damaging than in-phase loading, but traditional strain-based approaches resulted in non-conservative life predictions for out-of-phase loadings. Additionally, they reported that critical plane and plastic work approaches produced better life predictions than the others. Hoffman and Seeger [7] performed a local stress–strain analysis based on a generalized form of Neuber's rule. They concluded that the proposed procedure was able to accurately estimate the complete state of inelastic notch stresses and strains for proportional loadings.

Sakane et al. [8] studied notch effects on multiaxial low cycle fatigue using circumferentially grooved bars with three different notch root radii. Tests were performed in load control on a stainless steel as well as an alloy steel. They evaluated the use of Neuber's rule to estimate local strains at the notch root and concluded that it accurately estimated inelastic strains for shear deformation, but overestimated strains for tension deformation. Additionally, they found cycles to crack initiation to be significantly affected by notch geometry while there was little influence on crack propagation life. They also observed longer fatigue lives of notched torsion specimens as compared to notched tension specimens at the same equivalent nominal stress amplitude.

Atzori et al. [9] studied multiaxial fatigue of v-notched carbon steel specimens. Two different nominal load ratios ($R = 0$ and $-1$) were considered while the ratio of nominal axial to shear stress was kept equal to unity for all tests. They found a negligible effect on fatigue strength from changes between in-phase and out-of phase loadings. Additionally, an energy-based fatigue damage parameter, using first the total strain energy density at the notch tip and then an average strain energy density over a volume of material at the notch root, was used to correlate experimental fatigue data.

Gao et al. [10] studied fatigue behavior of sharp and blunt V-notched shafts made of 16MnR steel under tension, torsion, and in-phase and 90° out-of-phase axial–torsion loadings. They reported a significant notch size effect on fatigue life that increased with the number of loading cycles. FEA was used to derive local stresses and strains for use in fatigue life analysis. The combination of FEA results with two different critical plane damage parameters, Fatemi–Socie [11] and Jiang [12], provided predictions in excellent agreement with experimental fatigue lives for all loading cases.

Carpinteri et al. [13,14] studied multiaxial constant amplitude fatigue behavior of steel round bar specimens containing a drilled surface hole in the high-cycle fatigue regime for bending, torsion, and in-phase bending-torsion loadings. For combined loading tests, biaxiality ratios, $\lambda$, of 0, 1, 2, and infinity were considered. Using a critical plane defined based on the averaged direction of the principal stress axes and the ratio of smooth axial to smooth torsion fatigue limits, a non-linear combination of normal and shear stress quantities on this plane was used to compute the equivalent fatigue limit. Experimental results were in good agreement with the predicted values.

Susmel et al. [15] proposed a method for general multiaxial, constant or variable amplitude fatigue analysis based on the Modified Wöhler Curve Method and application of a critical distance approach for notched members. Stress quantities used in computation of the modified Wöhler curve were taken on a critical plane defined as the plane with the maximum variance of resolved shear stress. Experimental results for circumferentially grooved carbon steel specimens with three different notch root radii, subjected to three different load spectra, were shown to be in excellent agreement with the predicted fatigue lives. Cracks were observed to initiate on planes of maximum shear stress and propagate in mode I.

Firat [16] performed a numerical analysis of combined bending-torsion fatigue behavior of the SAE notched shaft specimens. The Smith–Watson–Topper [17] and Fatemi–Socie [11] critical plane approaches were evaluated in conjunction with the pseudo stress