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Reformulation in the frequency domain of a critical plane-based multiaxial fatigue criterion

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ABSTRACT

In the present paper, a new computationally-efficient frequency domain formulation of the critical plane-based Carpinteri–Spagnoli (C–S) criterion is proposed to evaluate the fatigue lives of smooth metallic structures subjected to multiaxial random loading. The critical plane orientation is here proposed to depend on the Power Spectral Density (PSD) matrix of the stress tensor. Then, the PSD function of an equivalent normal stress is defined by considering a linear combination of the PSD functions of the normal stress and the projected shear stress along the direction of maximum variance, with such stresses acting on the critical plane. Such an equivalent PSD function allows us to apply the Tovo–Benasciutti method to estimate the fatigue life of the structural components. The present frequency domain formulation of the C–S criterion is applied to some relevant fatigue tests related to smooth specimens under non-proportional bending and torsion random loading.

1. Introduction

Engineering structures prone to fatigue failure are often exposed to cyclic loading characterized by randomly varying amplitudes. The assessment of structural integrity, fatigue strength and reliability under random loading is a complex and critical issue in the design of such structures, which becomes even more complex in the case of multiaxial loading.

Despite the numerous research papers in the field, a correct quantification of the relationship between fatigue damage and load fluctuation features is still lacking, particularly when multiaxial random loadings are considered.

Various kind of procedures to assess the fatigue life of structural components under in-service multiaxial random loading are formulated in the time domain. They usually represent a generalization of their counterparts for constant amplitude loading, and usually require the knowledge of the time histories of the local stress or strain tensor components, the use of a counting procedure and a cumulative damage rule [1–7]. Experimental measurements or numerical simulations of the above time histories make such procedures costly and time-consuming, since many records are needed in order to obtain reliable statistical parameters of cycle distribution of random loading.

Following alternative procedures in frequency domain, strong efforts have been made in many research works to correlate fatigue damage with Power Spectral Density (PSD) characteristics of stress or strain components. Such procedures require that the loading process is known from a statistical point of view in terms of the PSD functions of the local stress or strain tensor components, as typically happens in the case (for instance) of random vibration tests [8–19].

In the present paper, an efficient frequency domain formulation of the Carpinteri–Spagnoli (C–S) criterion is proposed to evaluate the fatigue lives of smooth metallic structures subjected to multiaxial random loading.

The critical plane orientation, originally correlated to weighted mean directions of the principal stresses [20–24], is here assumed to be dependent on the Power Spectral Density matrix of the stress vector [25]. Then, the criterion presented in Refs. [6,7] for random loading is modified to evaluate the fatigue life by knowing the PSD function of an equivalent normal stress [14,25]. Accordingly, the shear stress vector acting on the critical plane is projected along the direction that maximizes the variance of such a stress (note that the projected shear stress obtained is time-varying in modulus, but its direction does not change with time), and the PSD function of the equivalent stress is defined by a linear combination of the PSD functions of the normal stress and the projected shear stress, both acting on the critical plane. The obtained equivalent PSD function allows us to apply the Tovo–Benasciutti method [18] in order to determine the fatigue life of the structural component being examined.

The frequency domain formulation of the C–S criterion is applied to some relevant random fatigue experimental results available in the literature [26], related to smooth specimens under non-proportional bending and torsion random loading.

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Nomenclature

$E[D_{NB}]$	expected fatigue damage per unit time by employing the narrow-band approximation
$E[D_{RC}]$	expected fatigue damage per unit time by employing the range-mean counting method
$E[D_{RFC}]$	expected fatigue damage per unit time by employing the rainflow counting method
$p_a(s)$	marginal probability distribution of the amplitude (s) of the $\{X(t)\}$ counted cycles
$p_p(X)$	probability distribution of peaks of $\{X(t)\}$
$PXYZ$	fixed frame
$PX'Y'Z'$	rotated coordinate system
$P\hat{1}\hat{2}\hat{3}$	coordinate system of the weighted mean principal stress axes
$Puvw$	coordinate system attached to the critical plane
$R_{X,X}(\tau)$	autocorrelation function of $\{X(t)\}$
$R_{i,j}(\tau)$	auto/cross-correlation functions of the i-th and the j-th stress vector components $s_i(t)$ and $s_j(t)$
$S_{eq}(\omega)$	equivalent PSD function
$\mathbf{s}_{xyz}(t)$	stress vector referred to the coordinate system $PXYZ$
$\mathbf{s}_{x'y'z'}(t)$	stress vector referred to the coordinate system $PX'Y'Z'$
$\mathbf{s}_{uvw}(t)$	stress vector referred to the coordinate system $Puvw$
$S_{X,X}(\omega)$	two-sided Power Spectral Density (PSD) function of $\{X(t)\}$
$\mathbf{S}_{xyz}\omega)$	Power Spectral Density (PSD) matrix of $\mathbf{s}_{xyz}(t)$
$S_{i,j}(\omega)$	coefficients of the $\mathbf{S}_{xyz}(\omega)$ matrix
$\mathbf{S}_{x'y'z'}(\omega)$	Power Spectral Density (PSD) matrix of $\mathbf{s}_{x'y'z'}(t)$
$S_{i',j'}(\omega)$	coefficients of the $\mathbf{S}_{x'y'z'}(\omega)$ matrix
$S_{3',3'}$	PSD function of the normal stress $\sigma_{z'}$
$S_{6',6'}$	PSD function of the shear stress $\tau_{y'z'}$
$\mathbf{S}_{uvw}(\omega)$	Power Spectral Density (PSD) matrix of $\mathbf{s}_{uvw}(t)$
$S_{3'',3'}$	PSD function of the normal stress σ_w
$S_{6'',6''}$	PSD function of the shear stress τ_{vw}
t	time
T	observation time interval
T_{cal}	calculated fatigue life
T_{exp}	experimental fatigue life
$\{X(t)\}$	one-dimensional ergodic stationary stochastic process
α_m	m-th bandwidth parameter, with m positive real number
α_2	regularity index
γ	rotation about w-axis
δ	angle between the averaged direction $\hat{\mathbf{1}}$ and the normal $\mathbf{w}$ to the critical plane (Fig. 3(b))
λ_m	m-th spectral moment, with m positive real number
μ_X	mean value of $\{X(t)\}$
ν_a	expected rate of occurrence of cycles of $\{X(t)\}$
$\nu^+_{\mu_X}$	expected rate of mean-upcrossings of $\{X(t)\}$
ν_p	expected rate of occurrence of peaks of $\{X(t)\}$
ν^+_0	expected rate of zero-upcrossings of $\sigma_{z'}$
$\sigma_{af,-1}$	normal stress fatigue limit for fully reversed normal stress (loading ratio $R = -1$)
σ^2_X	variance of the process $\{X(t)\}$
$\sigma^2_{\dot{X}}$	variance of the first time derivative of the process $\{X(t)\}$
$\sigma^2_{\ddot{X}}$	variance of the second time derivative of the process $\{X(t)\}$
$\tau_{af,-1}$	shear stress fatigue limit for fully reversed shear stress (loading ratio $R = -1$)
$\phi,\ \theta,\ \psi$	Euler angles
ω	pulsation

2. Basic properties of stochastic processes

Let us consider a one-dimensional ergodic stationary stochastic process $\{X(t)\}$, so that its statistical properties, invariant under a time shift, can be deduced from a single and sufficiently long record of such a process. Such ergodic and stationary assumptions are those commonly required for cyclic loading in order to handle fatigue calculations in the frequency domain (note that several load types, such as those due to traffic, wind, waves on civil engineering structures, can be treated as ergodic and stationary stochastic processes).

The above stochastic process is completely described, in the time-domain, by its two-sided Power Spectral Density (PSD) function, $S_{X,X}(\omega)$ [27]:

$$S_{X,X}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{X,X}(\tau)\, e^{-i\omega\tau}\, d\tau \tag{1}$$

defined as the Fourier transform of the autocorrelation function $R_{X,X}(\tau)$:

$$R_{X,X}(\tau) = E[X(t)\ X(t+\tau)] = \lim_{T\to\infty}\frac{1}{T}\int_0^T X(t)\ X(t+\tau)\, dt \tag{2}$$

where the operator $E[\cdot]$ indicates the expected value of a random variable, ω represents the pulsation, t and T are the time and the observation time interval, respectively (Fig. 1).

The spectral moments of the PSD function $S_{X,X}(\omega)$ are defined as follows [27]:

$$\lambda_m = \int_{-\infty}^{+\infty} |\omega|^m S_{X,X}(\omega)\, d\omega \tag{3}$$

where m is a positive real number. As is well known, there exist correlations between such moments and σ_X (variance of $\{X(t)\}$), $\sigma^2_{\dot{X}}$ and $\sigma^2_{\ddot{X}}$ (variances of $\{\dot{X}(t)\}$ and $\{\ddot{X}(t)\}$, which are the derivatives of the process $\{X(t)\}$):

$$\lambda_0 = \sigma^2_X \tag{4a}$$

$$\lambda_2 = \sigma^2_{\dot{X}} \tag{4b}$$

$$\lambda_4 = \sigma^2_{\ddot{X}} \tag{4c}$$

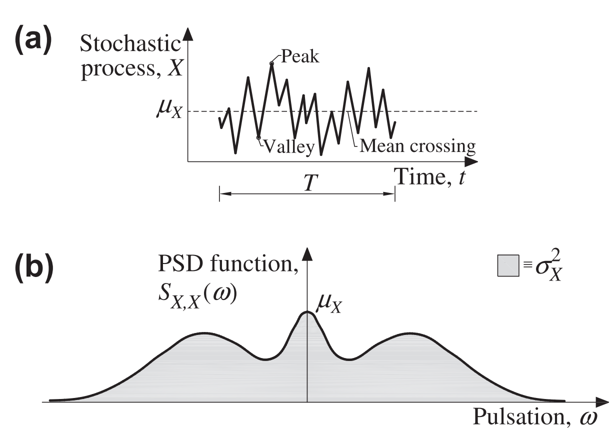

Fig. 1. Stochastic process: (a) characteristic parameters; and (b) related PSD function.