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Lifetime of semi-ductile materials through the critical plane approach

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ABSTRACT

This paper presents the results of fatigue strength estimation depending on the variable orientation of the critical plane for proportional and non-proportional bending and torsion with regard to specimens made of aluminium alloy 2017A. The algorithm applied for the fatigue strength evaluation is based on the Carpinteri-Spagnoli proposal and its subsequent modifications in accordance with the ideas developed by the authors. The objective of this paper is to search for a model which offers the best possible results of fatigue strength estimation for materials with properties which are intermediate between elastic-brittle and elastic–plastic, such as aluminium alloys, and a further insight into them with regard to their fatigue strength.

1. Introduction

The multiaxial fatigue criteria applied nowadays are predominantly based on the determination of an equivalent stress in the critical plane [1–9]. The orientation of the critical plane denotes the orientation of the surrounding of a material point in the space; however, it cannot be identified with the plane of the fatigue failure. The orientation of the critical plane and the location of fatigue failure plane are relative to the type of material used. The materials are often found in extremely different situations in the elastic-brittle and elastic–plastic states as well as intermediate properties between these states [10,11], as we can observe for aluminium alloys [8,12,13]. This paper focuses on the variable relations about the orientation of the critical plane with regard to fatigue strength for proportional and non-proportional bending and torsion. The determination of the critical plane is undertaken in accordance with a proposal by Carpinteri and Spagnoli for cyclic [1–4] and random [5] loading:

$$\beta = \frac{3}{2}\left[1 - \left(\frac{\tau_{af}}{\sigma_{af}}\right)^2\right]45^\circ \tag{1}$$

where β is the angle between the averaged direction of the maximum principal stress and the normal to the critical plane [2–4], whereas σ_{af} is the fully reversed normal stress fatigue limit and τ_{af} is the fully reversed shear stress fatigue limit. A description of the evolution of the above criterion can be found in Ref. [6].

The present paper makes several proposals which, in limit conditions, are pertinent to both elastic-brittle (hard metals $\sigma_{af}/\tau_{af} = 1$) and elastic–plastic materials (border line mild/hard metals $\sigma_{af}/\tau_{af} = \sqrt{3}$). As a result, this gives the angle 0° for σ_{af}/τ_{af} equal to 1 and 45° for σ_{af}/τ_{af} equal to $\sqrt{3}$. The boundary conditions are also fulfilled for instance through the following relationships:

$$\beta = \frac{9}{8}\left[1 - \left(\frac{\tau_{af}}{\sigma_{af}}\right)^4\right]45^\circ, \tag{2}$$

$$\beta = \frac{3\sqrt{3}}{3\sqrt{3}-1}\left[1 - \left(\frac{\tau_{af}}{\sigma_{af}}\right)^3\right]45^\circ, \tag{3}$$

$$\beta = \frac{3\sqrt{3}}{3\sqrt{3}-3}\left[1 - \left(\frac{\tau_{af}}{\sigma_{af}}\right)\right]45^\circ, \tag{4}$$

$$\beta = \frac{3}{(\sqrt{3}-1)^2}\left[1 - \left(\frac{\tau_{af}}{\sigma_{af}}\right)\right]^2 45^\circ. \tag{5}$$

Fig. 1 presents a graphical interpretation of Eqs. (1)–(5). One can note that, depending on the adopted relationship, the resulting angle changes: for instance, in the case $\sigma_{af}/\tau_{af} = 1.4$, the difference between calculated angles can be equal to as much as 16°, which affects the fatigue strength calculated by applying selected criteria of multiaxial fatigue [1,9,10]. In connection with this, it is necessary to derive an adequate expression on the basis of experiments.

The estimation of the fatigue strength in the critical plane is quite complex, and one criterion of multiaxial fatigue among all the criteria based on the concept of a critical plane must be selected.

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Nomenclature

b	fatigue strength exponent	α_η	angle which identifies the normal to the fracture plane
c	plastic strain exponent	β	angle between the averaged direction of the maximum principal stress and the normal stress
E	Young modulus	ε'_f	fatigue ductility coefficient
K'	coefficient of cyclic strength	λ	ratio of amplitudes of shear stress and normal stress
N	number of loading cycles	μ	variance, covariance
n'	exponent of cyclic hardening	σ_{eq}	equivalent stress
N_{ecp}	experimental number of loading cycles to failure	σ_η	normal stress related to the critical plane
N_{cal}	calculated number of loading cycles to failure	σ'_f	fatigue strength coefficient
N_f	number of loading cycles to failure	$\tau_{\eta s}$	shear stress on the critical plane
R_m	static tensile strength	v	Poisson ratio
t	time		
α	position of plane orientation with respect to axis of speciment		

The general expression of the equivalent stress in the critical plane can be presented as follows [9]

$$\sigma_{eq}(t) = B\tau_{\eta s}(t) + K\sigma_\eta(t), \tag{6}$$

where B and K depend on how the orientation of the critical plane is defined, $\tau_{\eta s}(t)$ is the history of the static stress and $\sigma_\eta(t)$ is the course of the normal stress in the critical plane:

$$\sigma_\eta(t) = \cos^2\alpha\sigma_{xx}(t) + \sin 2\alpha\tau_{xy}(t), \tag{7}$$

$$\tau_{\eta s}(t) = -\frac{1}{2}\sin 2\alpha\sigma_{xx}(t) + \cos 2\alpha\tau_{xy}. \tag{8}$$

where α defines the position of plane orientation with respect to axis of speciment.

The earlier analyses have been presented in [8] by using both the maximum variance of the normal stress and the maximum variance of the shear stress.

The method of the maximum variance of the normal stress can be expressed as follows:

$$\mu_{\sigma,\sigma}^{max} = \max_{\alpha_\eta}\left\{\mu_{\sigma,\sigma}(\alpha_\eta, t)\right\} = \max_{\alpha_\eta}\left\{\frac{1}{T_o}\int_0^{T_o}\sigma_\eta(\alpha_\eta, t)\sigma_\eta(\alpha_\eta, t)dt\right\}, \tag{9}$$

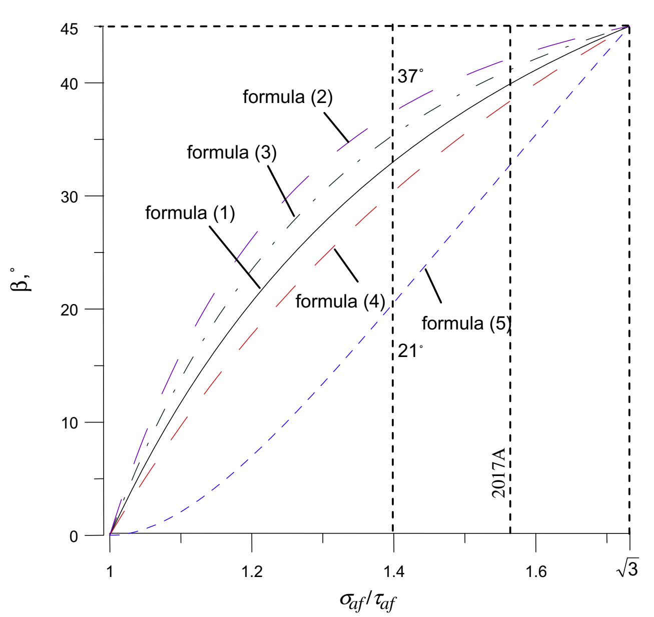

Fig. 1. Angle in relation to the maximum normal stress depending on the ratio of bending and torsional fatigue boundaries.

where α_η is the angle which identifies the normal to the fracture plane and T_0 is the observation time.

For the case of cyclic loadings, the observation time is equal to a single period, i.e. $T_0 = T$, and $\sigma_\eta(t)$ is the history of the normal stress oriented as the angle α in relation to the stress $\sigma_{xx}(t)$ for combined bending and torsion.

The method of the maximum variance of the shear stress can be expressed as follows:

$$\mu_{\tau,\tau}^{max} = \max_{\tau_{\eta s}}\left\{\mu_{\tau,\tau}(\alpha_{\eta s}, t)\right\} = \max_{\alpha_{\eta s}}\left\{\frac{1}{T_o}\int_0^{T_o}\tau_{\eta s}(\alpha_{\eta s}, t)\tau_{\eta s}(\alpha_{\eta s}, t)dt\right\} \tag{10}$$

while $\tau_{\eta s}(t)$ is the history of the shear stress as the angle α in relation to stress $\sigma_{xx}(t)$ for combined bending and torsion.

The search for the maximum variance of the shear and normal stresses in accordance with the respective expressions (9) and (10) provides the values of angles α_η and $\alpha_{\eta s}$, which identify the orientations of the critical planes defined by normal and shear stresses respectively.

By analysing the relationships in Eqs. (1)–(5), the expressions of the B and K coefficients in Eq. (6) are derived as depending on the Mohr's circle diameters ratio, k, at the fatigue limit for pure bending and pure torsion:

$$k = \frac{\sigma_{af}}{2\tau_{af}}. \tag{11}$$

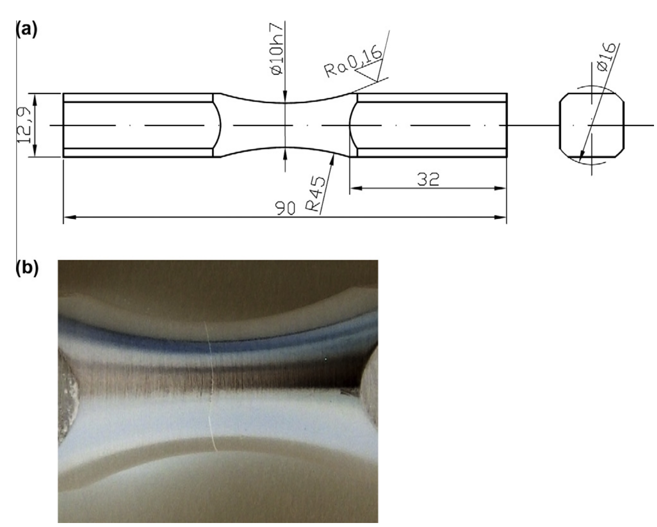

Fig. 2. (a) Geometry of specimens used in the testing and (b) specimen with an apparent fatigue crack.