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Multiaxial fatigue life estimation based on combined deviatoric strain amplitudes

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ABSTRACT

This contribution presents a model for multiaxial fatigue life estimation where a combination of deviatoric strain amplitudes defines the fatigue parameter. It also takes into account the influences of both hydrostatic stresses and mean deviatoric stresses. Assessment considers 211 proportional and non-proportional strain-controlled programs reported in the literature, including a number of cases with mean strains/stresses, for three steels and two aluminum alloys. The resulting life estimates correlates well with the experiments, falling in most cases within a factor of two bandwidth.

1. Introduction

Innovative solutions in the mechanical design process have demanded the development of new tools capable to provide more realistic descriptions of applied loads and material response. Such tools may contribute to "*get it right first time*" design by keeping experimental programs on prototypes (which can be time consuming and expensive) at a minimum scale. In this scenario, the simulations of more realistic multiaxial stress and strain histories have been possible with the use of currently available Finite Element softwares, together with more accurate evaluation of multiaxial fatigue damage (see [1–3] and references therein).

In the low cycle fatigue regime, many life estimates are based on critical plane approaches [3–6]. In these models, increase in normal stresses resulting from non-proportional hardening [7] has been considered to quantify the effect of non-proportional loading upon fatigue life [8–10]. Other approaches to fatigue damage include the damage evolution law proposed by Jiang [11], the short crack model of Vormwald and co-workers [12], the modified Coffin-Manson method by Susmel and co-workers [13], the damage mechanics approach by Carpinteri and co-workers [14,15], and the moment of inertia method by Meggiolaro and Castro [16] among others.

As an alternative to the currently available approaches, this paper proposes a model for fatigue life estimation, which presents, as its main feature, a measure of combined deviatoric strain amplitudes based on the concept of prismatic hull [17–19]. The influence of normal stresses on fatigue life is taken into account in terms of the amplitude and the mean value of the hydrostatic stress. Both strain and hydrostatic stress terms incorporate the effect of non-proportional loading upon fatigue life. Further, the mean value of the second invariant of the deviatoric stresses is included in order to take into account the influence of mean shear stresses upon fatigue life. The resulting model produces fatigue life estimations which correlate well – within a factor of two bandwidth – with experimental data available in the literature for steels and aluminum alloys subjected to proportional and non-proportional, synchronous and asynchronous, axial–torsional strain histories.

2. Fatigue model

This section starts with the definition of a measure of strain amplitude (within the setting of multiaxial loading histories), assumed in this study as one of the most important driving forces responsible for crack nucleation in materials undergoing fatigue failure in shear mode. The close relation between fatigue damage and cyclic plasticity in metals (where plastic state variables are usually written in terms of deviatoric quantities) provides the basic motivation for considering the deviatoric strains in the definition of the fatigue parameter proposed in this paper. Further, it is assumed that, under multiaxial loading, fatigue degradation is due to a combination of loading modes, described here in terms of strain histories in the deviatoric space.

If ε denotes the *total strain tensor*, then the *deviatoric strain tensor* $\boldsymbol{e}$ is given by:

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$$\boldsymbol{e} = \boldsymbol{\varepsilon} - \frac{1}{3} tr(\boldsymbol{\varepsilon})\boldsymbol{I} \tag{1}$$

and, for the purpose of this study, it can be written in terms of an orthonormal basis $\{\boldsymbol{N}_i;\ tr(\boldsymbol{N}_i) = 0, i = 1, \ldots, 5\}$ as follows:

$$\boldsymbol{e} = \sum_{i=1}^{5} e_i \boldsymbol{N}_i = \sum_{i=1}^{5} (\boldsymbol{e} : \boldsymbol{N}_i)\boldsymbol{N}_i \tag{2}$$

where the projection $\boldsymbol{e}$: $\boldsymbol{N}_i$ of the deviatoric tensor $\boldsymbol{e}$ onto each deviatoric basis element $\boldsymbol{N}_i$ produces the corresponding coefficient e_i, $i = 1, \ldots, 5$. In particular, for the arbitrarily chosen orthonormal basis

$$\boldsymbol{N}_1 = \frac{1}{\sqrt{6}}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \boldsymbol{N}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$
$$\boldsymbol{N}_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \boldsymbol{N}_4 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \boldsymbol{N}_5 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{3}$$

the coefficients of the deviatoric strain tensor assume the following forms:

$$e_1 = \frac{1}{\sqrt{6}}(2\varepsilon_x - \varepsilon_y - \varepsilon_z), \quad e_2 = \frac{1}{\sqrt{2}}(\varepsilon_y - \varepsilon_z),$$
$$e_3 = \frac{1}{\sqrt{2}}\gamma_{xy}, \quad e_4 = \frac{1}{\sqrt{2}}\gamma_{xz}, \quad e_5 = \frac{1}{\sqrt{2}}\gamma_{yz}. \tag{4}$$

where ε_x, ε_y and ε_z are normal strains, whereas γ_{xy}, γ_{xz} and γ_{yz} are shear strains components with respect to a Cartesian coordinate system.

It follows that the history of deviatoric strains along a multiaxial loading can be described in vector form as follows:

$$e(t) = [\, e_1(t) \quad e_2(t) \quad e_3(t) \quad e_4(t) \quad e_5(t)\,]^T, \tag{5}$$

and represented as a curve in $\mathbb{R}^5$. Each strain component $e_i(t)$, $i = 1, \ldots, 5$, defines a history for which we can compute the corresponding amplitude:

$$a_i = \frac{1}{2}(\max_t\ e_i(t) - \min_t\ e_i(t)), \quad i = 1, \ldots, 5 \tag{6}$$

as is illustrated in Fig. 1.a for a three-axial example.

In the present work, fatigue degradation is assumed to result from a *combination of loading modes*, described within this setting by combining the strain amplitudes a_i, $i = 1, \ldots, 5$ as follows:

$$\gamma = \left(2\sum_{i=1}^{5} a_i^2\right)^{\frac{1}{2}}. \tag{7}$$

It is claimed here that the maximum value produced by Eq. (7) – amongst all orientations Θ of the frame describing the strain history in $\mathbb{R}^5$-is the appropriate measure to quantify the contribution of the strain history to the fatigue damage. In this setting, the *combined amplitude of the deviatoric strain* is defined as

$$\gamma_{\text{dev}} = \max_{\Theta}\left(2\sum_{i=1}^{5} a_i^2(\Theta)\right)^{\frac{1}{2}} \tag{8}$$

where, for each orientation Θ, the amplitudes $a_i(\Theta)$ can be computed from the strain histories $e_i(\Theta)$ in the Θ-oriented frame (see Fig. 1b for a three-axial example):

$$a_i(\Theta) = \frac{1}{2}(\max_t e_i(\Theta, t) - \min_t e_i(\Theta, t)), \quad i = 1, \ldots, 5. \tag{9}$$

It is worthwhile noticing that the maximum in Eq. (8) ensures the frame independence of the proposed measure. The factor two multiplying the sum within the square root in Eq. (8) makes the resulting measure γ_{dev} be equal to the shear strain amplitude in the case of completely reversed simple shear loading history.

In order to describe frame orientations Θ in Eqs. (8) and (9), the so-called Givens rotations (often employed in the context of diagonalization techniques for symmetric matrices (see [20], for instance)) can be considered: rotations in two-dimensional subspaces defined by axes 1–2, 1–3,..., 4–5 are performed as

$$e(\Theta, t) = \mathbf{Q}_{45}\mathbf{Q}_{35} \ldots \mathbf{Q}_{12}e(t). \tag{10}$$

where matrices $\mathbf{Q}_{pq}$, $p, q = 1, \ldots, 5, p \neq q$, in Eq. (10) are rotation matrices in plane p–q, built from a five-dimensional identity matrix by replacing its pp-th and qq-th elements with $\cos\theta_{pq}$, its pq-th element with $\sin\theta_{pq}$ and its qp-th element with $-\sin\theta_{pq}$. For instance, a rotation θ_{35} in plane 3–5 is represented by the following matrix:

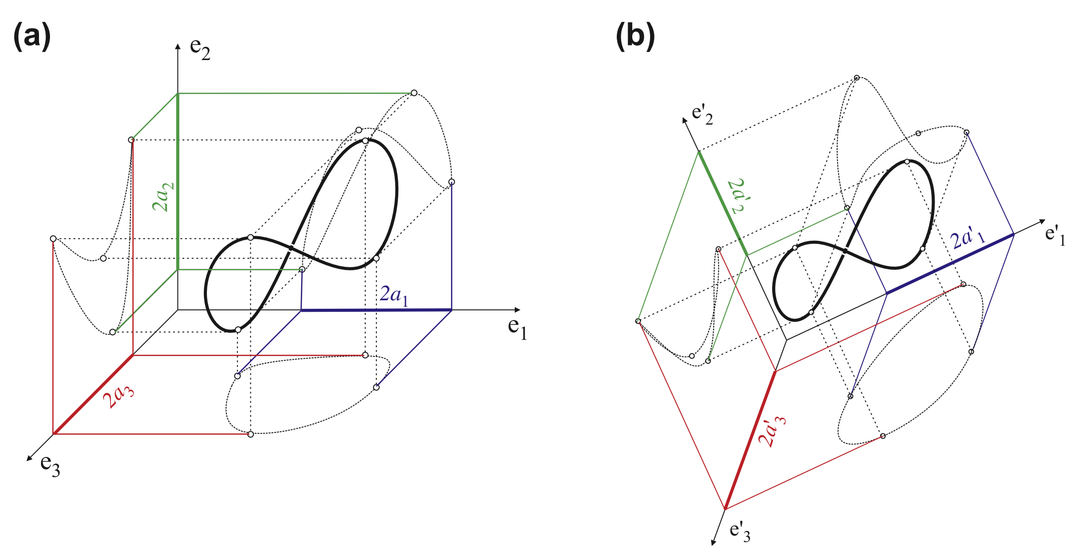

Fig. 1. (a) Description of strain history in terms of the coefficients resulting from projection upon basis elements of the deviatoric space; (b) deviatoric strain amplitudes along arbitrarily rotated axes.