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Automatic modelling of cohesive crack propagation in concrete using polygon scaled boundary finite elements

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ABSTRACT

An automatic cohesive crack propagation modelling methodology for quasi-brittle materials using polygon elements is presented. Each polygon is treated as a subdomain that is modelled by the scaled boundary finite element method (SBFEM). Generalised stress intensity factors (SIFs) based on matrix power function solutions of singular stress fields obtained from the SBFEM following standard finite element stress recovery procedures is used to evaluate the crack propagation criterion and determine the crack propagation direction. Interface elements model the fracture process zones and are automatically inserted into the polygon mesh as the crack propagates. A shadow domain procedure couples the polygons and interface elements. It computes the load–displacement response and crack propagation criterion, taking into account the cohesive tractions on the crack edges that are modelled as side-face tractions in the SBFEM. Cracks are propagated using a simple, yet flexible local remeshing procedure that can remesh any arbitrary polygon. Only minimal changes are made to the global mesh structure each time the remeshing algorithm is called. Five cohesive crack propagation benchmarks are modelled to validate the developed method and demonstrate its salient features.

1. Introduction

Fracture in quasi-brittle materials such as concrete involves a process zone in which normal and shear tractions can be transferred across the crack surfaces due to aggregate interlocking and surface friction. In order to capture the physically observed cracking phenomenon the process zone must be considered in the numerical models/simulations. The cohesive zone model of Hillerborg et al. [1] is most commonly used to model the process zone. In the finite element method (FEM), the process zone is usually modelled using zero thickness interface elements.

Different modelling strategies with interface elements have been proposed in the literature such as pre-inserting interface elements along known crack paths, e.g. [2–4], pre-inserting interface elements along all element interfaces in the mesh, e.g. [5–7] and automatically inserting interfaces elements along the crack surfaces as the crack propagates, e.g. [8–11]. The methods developed in [2–4] require a priori information of the crack paths that are usually determined from experiments. The methods developed in [8–10] require sophisticated remeshing algorithms to propagate the crack in addition to high mesh densities or special finite elements, e.g. [12,13] to model the singular stress fields around crack tips. Although the methods developed in [5–7,11] do not require remeshing, high mesh densities are required so that the predicted crack paths are smooth and accurate.

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Nomenclature

A	area
$\mathbf{c}$	integration constants
$\mathbf{D}$	constitutive material matrix
E	Young's modulus
$\mathbf{I}$	identity matrix
$\mathbf{k}$	stiffness components of traction–softening curves
$\mathbf{K}$	stiffness matrix of polygon elements
L	characteristic length
$\mathbf{N}$	shape function matrix
$\mathbf{q}$	internal nodal forces along radial line
$\mathbf{R}$	rotation matrix
s	crack sliding displacement
$\mathbf{S}$	matrix of real eigenvalues
$\mathbf{u}$	displacement vector
w	crack opening displacement
$\mathbf{Z}$	Hamiltonian coefficient matrix
Γ	boundary of domain
η	circumferential coordinate
θ	angle
λ	eigenvalues
ν	Poisson's ratio
ξ	radial coordinate
σ	stress
$\mathbf{\Psi}$	transformation matrix
Ω	domain
$\mathbf{B}_i$	strain displacement matrices
$\mathbf{c}^{(s)}$	integration constants corresponding to singular stress modes
$\mathbf{E}_i$	coefficient matrices
f_c	compressive strength
f_t	tensile strength
$\mathbf{F}_t$	nodal load vector of side-face forces
G_f	fracture energy
$\mathbf{K}(\theta)$	vector of generalised stress intensity factors
N_g	number of Gaussian points
$\hat{r}$	polar radial coordinate
$\mathbf{S}^{(n)}$	diagonal block of negative eigenvalues
$\mathbf{S}^{(s)}$	matrix of orders of singularities
$\mathbf{u}_b$	nodal displacements at boundary
$\mathbf{\Psi}^{(q)}$	modal forces
$\mathbf{\Psi}^{(u)}$	modal displacements
$\mathbf{\Psi}_\sigma$	stress mode
$\mathbf{\Psi}_\sigma^{(s)}$	singular stress modes
$\mathbf{\Psi}_{\sigma L}^{(s)}$	singular stress modes at characteristic length, L

To tackle the difficulties in remeshing encountered with FEM in crack propagation modelling, nodal enrichment techniques such as the extended FEM (XFEM) [14] and embedded crack models [15] have been proposed. In these methods, the nodes or elements that are cut by the crack path are enriched with discontinuous Heaviside function to model cracks. The XFEM includes additional nodal enrichment with singular stress functions to model singular stress fields around cracks. Both XFEM and embedded crack models share the same appealing feature in that they do not require remeshing to model crack propagation. To implement cohesive crack models in the XFEM and embedded crack models, the governing equations are augmented to incorporate the work done by the cohesive tractions along the crack edges. Many applications of cohesive crack propagation for elastostatics and elastodynamics with XFEM [16–19] and embedded crack models [20–22] have been reported in the literature. The concepts embodied in these nodal enrichment techniques have also been implemented in meshless methods to model cohesive crack propagation [23,24].

Recently polygon based finite elements such as polytope elements [25–27] that are based on barycentric coordinates shape functions [28], the n-sided polygonal smoothed FEM (SFEM) [29], and the Voronoi cell FEM (VCFEM) [30,31] have been proposed for modelling problems in elastostatics. Some of these methods have been implemented together with nodal