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Stress intensity factor analysis of a three-dimensional interface crack between dissimilar anisotropic materials under thermal stress

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ABSTRACT

A numerical method for evaluating the stress intensity factors (SIFs) of a three-dimensional interface crack between dissimilar anisotropic materials subjected to thermal and mechanical stresses is proposed. The M_1-integral method was applied to an interfacial crack between three-dimensional anisotropic bimaterials under thermal stress. The moving least square approximation was utilized to calculate the value of the M_1-integral. The M_1-integral in conjunction with the moving least square approximation can be used to calculate the SIFs from nodal displacements obtained by finite element analysis. SIF analyses were performed for double edge cracks in jointed dissimilar isotropic semi-infinite plates subjected to thermal load. Excellent agreement was achieved between the numerical results obtained by the present method and the exact solution. In addition, we computed the SIFs of an external circular interfacial crack in jointed dissimilar anisotropic solids under thermal stress and showed the distributions of SIFs along the crack front. The distribution of stress and the crack opening displacement obtained by the asymptotic solution using the computed SIFs were compared with those obtained by the finite element analysis with fine mesh. They were almost identical to each other, except for the minor component of SIFs that is much smaller than the major component of SIFs. These results indirectly demonstrate the accuracy of the obtained SIFs.

1. Introduction

Electronic devices and micro-electro-mechanical systems (MEMSs) are composed of many different materials and have many interfaces. The thermal stress resulting from the mismatch of the coefficient of thermal expansion between different materials often causes delamination along the interface. The delamination of the interface between different materials is one of the main causes of the malfunction or the reliability degradation of electronic devices.

The stress intensity factors (SIFs) of an interface crack are important for evaluating the fracture at interfaces. Gotoh [1], Clements [2], Willis [3], Bassani and Qu [4], and Wu [5] have investigated the stress field around interfacial cracks between dissimilar anisotropic materials. Hwu [6] has proposed an asymptotic solution for stress around an interface crack between dissimilar anisotropic materials using the Stroh formalism [7], and defined the SIFs of an interface crack. Few analytical solutions of the SIFs of interface cracks between dissimilar anisotropic materials have been proposed due to the mathematical difficulty. The energy release rate of an interfacial crack can be obtained using an energy method such as the virtual crack extension method, the *J*-integral method or the crack closure integral method. Sun and Qian [8] and Ikeda et al. [9] proposed

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Nomenclatures

A and **B** Stroh's eigenvectors matrices of a material
C_{ijks} elastic stiffness tensors
D and **W** bi-materials constant matrices
E bi-materials constant matrix
E_{ij} bi-materials constant tensors
f function vector
G energy release rate
J J-integral
$\mathbf{K} = \begin{Bmatrix} K_{II} \\ K_{I} \\ K_{III} \end{Bmatrix}$ stress intensity factors
L and **S** Barnett–Lothe tensors
l_k characteristic length
L_c small segment along a crack front
$\mathbf{M}^*$ bi-materials matrix
n_i unit normal vector on the boundary
$\mathbf{N}_1$ and $\mathbf{N}_3$ functions of elastic constants
p linear basis
q_k (k = 1, 2, 3) test functions
r distance from the crack-tip
S_o and S_t surfaces of Tubular domain V
t_i boundary tractions
u displacement vector
u_i (i = 1, 2, 3) displacements
V tubular domain surrounding the crack segment
W^e elastic strain energy density
w weight function
x_i (i = 1, 2, 3) rectangular coordinate system
α_{ij} (i = 1, 2, 3; j = 1, 2,3) coefficients of thermal expansion tensors
δ_α (α = 1, 2, 3) eigenvalues
ε oscillation index
η virtual crack advance
ϕ shape function
$\mathbf{\Lambda}$ eigenvector matrix
$\boldsymbol{\lambda_i}$ (i = 1, 2, 3) eignevectors
ε_{ij} (i = 1, 2, 3; j = 1, 2,3) elastic strain tensors
$\mathbf{\Gamma}$ arbitrary contour pass on the $x_1 - x_2$ plane enclosing the crack tip
μ_k shear modulus
σ_{ij} (i = 1, 2, 3; j = 1, 2,3) stress tensor
ϑ temperature
ξ_k (k = 1, 2, 3) crack advanced vectors
$\mathbf{\Omega}$ area surrounded by the contour pass $\mathbf{\Gamma}$
$\boldsymbol{\psi}$ stress function

numerical methods to calculate the mode-separated SIFs of a two-dimensional interfacial crack between dissimilar anisotropic materials subjected to mechanical loads. Nagai et al. [10] and Nomura et al. [11] presented computational methods to determine the SIFs of a two-dimensional interfacial crack and corner between dissimilar anisotropic solids under thermal stress, respectively. Nagai et al. [12] proposed the modified M_1-integral method, which is a combination of the J-integral method and the superposition method, to obtain the SIFs of a three-dimensional crack between anisotropic bimaterials. However, there is still no numerical method based on the energy method to determine the SIFs of a three-dimensional interfacial crack between dissimilar anisotropic materials subjected to thermal stress.

We extended the M_1-integral to determine the SIFs of a three-dimensional interface crack between dissimilar anisotropic materials under thermal stress. In this method, the moving least-square approximation is employed to calculate the value of the M_1-integral, and the stress and strain in the M_1-integral are approximated from the nodal displacements obtained by the finite element analysis. Therefore, in the computation of the M_1-integral, this method does not need to extract any elemental data from the finite element analysis. The M_1-integral method presented here can calculate the SIFs easily and requires less time for the data preparation than other procedures.