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Fourier basis for the engineering assessment of cracks in residual stress fields

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ABSTRACT

A theoretical basis is presented for determining the significance of a residual stress distribution of arbitrary shape on the crack tip stress intensity factor for a centre-cracked plate as a function of crack length. The Fourier series based approach enables one to increasingly add more spatial definition to the stress field and thereby determine the level of detailed knowledge of the residual stress required to make a reliable assessment of structural integrity. The approach is applied to examples of measured symmetric distributions of residual stresses in welded plates and used to determine the significance of residual stress length-scales in fracture mechanics analysis.

1. Introduction

A structure can be defined as any assembly of materials intended to withstand loads. Engineering structural integrity assessment is concerned with design against premature failure, but structures must also be fit for purpose and economic to make. Residual stresses can exist inside a body or structure in the absence of any externally applied loads at various length-scales [1]. As such they add to (or subtract from) applied stresses and are often the cause of unexpected failure. The science of fracture mechanics [2] has developed rapidly since the mid-1950s and provides methods [3,4] for engineers to assess whether flaws or crack-like defects in structures will progress through various mechanisms, including corrosion, creep and fatigue, and ultimately become unstable. Fracture mechanics studies involve both stress analysis aspects and the resistance behaviour of the material to the stresses imposed. The present study is based on the linear elastic analysis of stress surrounding a crack in an ideal material having isotropic elastic properties, which is an essential ingredient in most current fracture assessment methods. Here we lay out a new approach based on Fourier series analysis, that formalises the consideration of length-scale initiated by Bouchard and Withers [5] and first quantified in a specific case in [6]. It has certain advantages over established approaches. First it provides a general stress intensity factor solution that can be applied to arbitrary residual stress profiles. Secondly, it enables one to increasingly add more spatial detail to the stress field and thereby determine the level of detailed knowledge of the residual stress required to make a reliable fracture assessment for a crack of a given length. This has consequences both for the measurement strategy and the complexity of computational methods used to characterise residual stresses, for example in welded structures.

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Nomenclature

a	half-length of an embedded through-thickness crack in a wide plate (m)
b	half-length across which a symmetric periodic distribution of residual stress applies (m)
c	x co-ordinate where a self-equilibrated residual stress profile changes sign (m)
f	equivalent force acting at the centroid of a planar intersection sub-domain (N)
m	equivalent moment acting around an axis passing through the centroid of a planar sub-domain (Nm)
n	order of term in a Fourier series expansion
p	maximum order of cosine or sine terms included in a Fourier series expansion
r	index term
x, y, z	Cartesian axes with the z-axis directed out of the domain plane (m)
A_n	Fourier series cosine term coefficient for order n
B_n	Fourier series sine term coefficient for order n
E_p	bounding error given by truncating a Fourier series expansion for K_I at $n = p$
F	equivalent force acting at the centroid of a planar intersection domain (N)
G	approximate K_I solution function for a cosine residual stress field
J_0	Bessel function of the first kind and order zero
K_I	stress intensity factor under mode I loading, (MPa $\sqrt{m}$)
M	equivalent moment acting around an axis passing through the centroid of a planar domain (Nm)
SIF	stress intensity factor (MPa $\sqrt{m}$)
λ	wavelength of a self-equilibrated residual stress distribution (m)
σ_{ij}	stress in direction i acting on the plane with the normal in the direction j (MPa)
σ_0	reference value of a residual stress field (MPa)
σ_∞	remote uniform stress in an infinitely wide plate (MPa)
σ_b	bending component of stress that spans a defined sub-domain (MPa)
σ_m	membrane component of stress that spans a defined sub-domain (MPa)
σ_{se}	self-equilibrated component of stress that spans a defined sub-domain (MPa)

"Fitness-for-service" life and integrity assessment procedures for engineering structures [7–9], explicitly concede the presence of actual, or potential, cracks in welded joints or other critical locations and consider the effect of residual stress at various levels of detail:

i. The simplest residual stress characterisation approach, widely adopted, is to assume the presence of a uniform residual stress equal in magnitude to the material yield strength [7,9]. This always produces an overestimate of the local stress field at the crack tip, characterised by the stress intensity factor (SIF).
ii. An upper-bound linear residual stress profile has been advocated for determining SIFs of cracks at T-butt welds [10]. This gives an overestimate of the SIF for cracks in the tensile stress region but ignores the self-equilibrating nature of the stress field.
iii. Non-linear residual stress profiles that provide an upper-bound to measured data for a generic type of weldment can be found in published compendia [7–9] and are often used. This approach also secures an overestimate of the SIF, especially for longer cracks.
iv. A more realistic residual stress profile is assumed for a generic type of weldment based on analytical modelling and/or numerical analysis and/or measurements [11].
v. A best estimate residual stress distribution is considered for a specific weldment based on numerical analysis validated by diverse measurements [7].

Clearly the level of information required about the stress field increases moving down this list, as does the effort needed to calculate the resulting SIF as a function of crack length. Case (iv) relies on analytical modelling and measurements, for example [11,12]. Case (v) requires application of non-linear computational mechanics methods to predict residual stresses introduced by manufacturing processes [13] and, for safety–critical applications, validation of the results through application of diverse measurements [7]. Modern measurement techniques [1,14] have the potential to give a reliable characterisation of the true state of stress in a real structure, but the information is invariably incomplete; that is only some components of the stress tensor are measured to a finite length-scale resolution over a limited spatial sub-domain. For example strain gauges provide surface measurements at a few key locations; neutron diffraction measurements tend to be made on line-scans and synchrotron X-ray measurements can feasibly be made over areas, for example [15]. Further, the measured data have associated scatter and uncertainties [16]. A key issue for both computational modelling and measurement is what level of residual stress detail is required; this question can be conveniently studied in terms of self-equilibrated stress wavelength analysis. Here we develop a Fourier approach to examine how much detail of the residual stress field is required to get a sufficient estimate of the stress intensity factor for a given crack length. Further, the approach is completely general providing a simple means of determining the stress intensity factor for an arbitrary stress field along the proposed crack path.