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Strip yield crack analysis for multiple site damage in infinite and finite panels – A weight function approach

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ABSTRACT

An analytical approach to the Dugdale strip yield model for multiple site damage is presented by using the weight function method. Two example problems, an array of periodic collinear cracks in an infinite sheet and a coalesced center crack in a finite width panel, are analyzed by the closed-form weigh function; the effect of finite boundary is considered. Results are extensively verified against available analytical and numerical solutions. The capability of the closed-form weight function for the strip yield model analysis of multiple site damage is demonstrated.

1. Introduction

Aircraft structural integrity has been a great concern since the 1950s, when fatigue and fracture were recognized as big threat to the safe operation in the air. Present aircraft structures are designed according to the fracture mechanics based damage tolerance principle in order to prevent catastrophic structural failure caused by crack like defects. However, a special type of cracking problem, the multiple site damage (MSD) did not come to light until 1988 when a large piece of the pressure cabin skin (5.5 m long) of upper fuselage of Aloha Airlines Boeing 737 was ripped off in the mid-air over Hawaii [1]. This dramatic accident by a sudden linking-up of a large number of small cracks at the rivet holes in the same row had prompted many research activities to address the MSD issue, and all the major commercial airplane manufacturers were required to evaluate their aircraft for MSD in the critical areas of the wing, empennage, and pressure fuselage.

MSD is one of the two important sources for widespread fatigue damage (WFD). It is characterized by the simultaneous presence of fatigue cracks in the same structural element [2–4]. Reductions in residual strength due to MSD are typically 20–40% but even values up to 50% have been reported [5]. MSD is a complex phenomenon that is very difficult to analyze with standard methods. Past efforts have been mostly relying on advanced numerical techniques, especially the finite element method. Analytical approaches, such as the complex stress function formulation and integral equations, have been successfully applied to the assessment of MSD for simple geometrical configurations. The Dugdale strip yield model [6] was found to be a useful tool to tackle the MSD problem [9,11,12]. The model has been explored by a number of researchers. Nilsson and Hutchinson proposed a modification of Dugdale strip yield model using a damage-reduced yield strength in the plastic zone [7]. Subsequent studies on this model were conducted by Jeong and Brewer [8], Collins and Cartwright [9,10], Cherry and

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Nomenclature

$2a$	crack length
d	segment length
E	Young's modulus
f	non-dimensional stress intensity factor
K	stress intensity factor
P	point forces (unit thickness)
r	plastic zone size
W	panel half-width
x, y	Cartesian coordinates
ν	Poisson's ratio
σ	remote uniform stress
$2a_0$	physical crack length
$2a_1$, $2a_2$	physical crack length of three collinear cracks in a finite width panel
d_1, d_2	segment interval
f_{seg}	non-dimensional stress intensity factor for segment uniform pressure load
f_{rivet}	non-dimensional stress intensity factor for riveted load
σ_f	flow strength
σ_s	yield strength
σ_{ult}	Ultimate strength
E'	effective Young's modulus, E for plane stress or $E/(1-\nu^2)$ for plane strain
ν'	effective Poisson's ratio, ν for plane stress or $\nu/(1-\nu)$ for plane strain
$\ln(x)$	the natural logarithm of the elements of x
$m(a, x)$	weight function
$\sec(\pi a/2)$	the secant of the elements of $\pi a/2$
$f_r(a)$	non-dimensional stress intensity factor for reference load case

Mall [11], Nilsson [12], and Nishimura [13]. However, these analyses were limited to idealized geometries, mainly the infinite sheet, and the effect of finite boundary was ignored. For real aircraft structural parts, neglecting the size effect will lead to significant error in the residual strength estimation. Therefore, reliable and efficient analytical methods that are able to handle MSD problems with finite boundaries are very desirable, especially for parametric studies.

The Dugdale strip yield model is essentially a superposition of two elastic crack problems, one of which involving a segment of uniformly distributed stress in the plastic zone. These crack problems can be easily solved by the weight function method, for which the load complexity and the finite geometry do not pose any difficulties. However, to the writer's knowledge, the use of weight functions in MSD analysis has not yet been explored so far. The purpose of this paper is, therefore, to make a first attempt to the MSD analysis by weight function method. Results from the closed-form weight function method are extensively verified against available analytical and numerical solutions where possible, and the effect of finite boundary has been taken into account. The very favorable comparisons do give strong support to the weight function approach to the MSD problems.

2. Formulation of the problem

We consider two multiple site damage cases: (i) an array of collinear periodic cracks in an infinite sheet, Fig. 1 and (ii) one large center crack formed by coalescing three un-equal length center cracks in a panel of finite width, with compressive yield stress uniformly acting along the un-cracked ligament and in the crack tip region, Fig. 2. The external load for both cases is uniform remote tensile stress. For the periodic collinear crack case, rivet force loading is also considered.

For the first case in which the plastic zones are separated, Fig. 1, the analysis is carried out by assuming each crack (including the crack tip plastic zones) as a fictitious crack, and formulating the problem based on the vanishing singularity at the fictitious crack tips. For the second case in which the plastic zone are coalesced, the analysis is conducted by assuming the coalesced three un-equal length cracks as one single fictitious crack subjected to segment pressure distribution in the un-cracked ligament and crack tip region, in addition to the applied external load, Fig. 2. Fig. 3 shows three other loaded cases for collinear periodic cracks in an infinite sheet.

3. Weight function method for crack problem analysis

The weight function method is a versatile, reliable and efficient method for crack problem analysis, especially for cracks subjected to complex loadings. The method was first proposed by Bueckner [14], and further advancements were made by