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Weight function based Dugdale model for mixed-mode crack problems with arbitrary crack surface tractions

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ABSTRACT

Weight function theory states crack surface displacements can be found for any arbitrary distribution of mode I, or mixed-mode crack face tractions via that geometry's weight functions. This statement is validated via finite element analysis of the infinite center-cracked plate for various mixed mode loadings. An elastic-perfectly plastic material is considered using a Dugdale approach and compared to elastic–plastic finite element simulations. The weight function method in all cases agrees well with the finite element simulations for small scale yielding at the crack tip. As the maximum traction value approaches one-half the yield strength discrepancies become larger due to violation of small scale yielding.

1. Introduction

The strip-yield model, developed by Dugdale [1] and Barenblatt [2] assumes a crack tip plastic zone ρ collinear with the crack a under plane stress in a non-hardening material. The model superposes two elastic problems: a crack of effective length $a+\rho$ under an applied traction over the entire length and a crack of effective crack of length $a+\rho$ under a cohesive zone closure stress acting over the length ρ. Because the stresses in the plastic zone must be finite, the plastic zone length ρ is chosen so the stress intensity factors from the remote and cohesive zone loading cancel one another at the crack tip. Fig. 1 is a strip-yield model schematic showing the crack, plastic zone, remote loading and closure loading.

Using a weight function methodology, the strip-yield model can be generalized to find crack face displacements for arbitrary distributions of crack face tractions. The literature shows that weight functions allow treatment of any geometry under arbitrary mode I loading [3–13] and can be used to find stress intensity factors under mixed mode loading [14–22]. However, a straightforward validation of the weight function based Dudale model's ability to provide crack surface displacements under arbitrary distributions of mixed-mode crack face tractions is not available. This paper documents such an investigation. An infinite center-cracked plate, a geometry for which an analytical weight function exists [23] was chosen for the study. The mentioned crack surface displacements are needed for crack closure simulations via the modified strip-yield model.

2. Weight functions

Weight functions, first introduced by Bueckner [24] and Rice [25], are a property of a cracked geometry and independent of the loading. Once a particular loading is used to derive a geometry's weight function, the stress intensity factors and crack face displacements can be found for any other loading [23].

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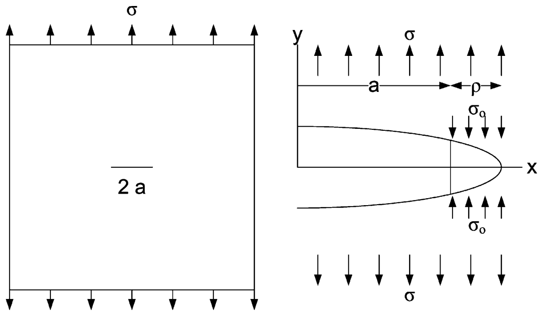

Fig. 1. Dugdale strip yield schematic.

Using the weight function approach and considering a Dugdale crack, the modes I and II crack surface displacements $\delta_{\mathrm{I}}(x)$ and $\delta_{\mathrm{II}}(x)$ can be calculated as follows:

$$\delta_I(x) = \frac{2}{E}\int_x^{a+\rho} K_{\mathrm{I}}(\xi)m_{\mathrm{I}}(x,\xi)d\xi \tag{1}$$

$$K_{\mathrm{I}}(\xi) = \int_0^{\xi} \sigma(x)m_{\mathrm{I}}(x,\xi)dx \quad \text{if } x < \xi \tag{2}$$

$$K_{\mathrm{I}}(\xi) = \int_0^{\xi} \sigma(x)m_{\mathrm{I}}(x,\xi)dx - \int_0^{\xi} \sigma_o m_{\mathrm{I}}(x,\xi)dx \quad \text{if } x > \xi \tag{3}$$

$$\delta_{\mathrm{II}}(x) = \frac{2}{E}\int_x^{a+\rho} K_{\mathrm{II}}(\xi)m_{\mathrm{II}}(x,\xi)d\xi \tag{4}$$

$$K_{\mathrm{II}}(\xi) = \int_0^{\xi} \sigma(x)m_{\mathrm{II}}(x,\xi)dx \quad \text{if } x < \xi \tag{5}$$

$$K_{\mathrm{II}}(\xi) = \int_0^{\xi} \tau(x)m_{\mathrm{II}}(x,\xi)dx - \int_a^{\xi} \tau_0 m_{\mathrm{II}}(x,\xi)dx \quad \text{if } x > \xi \tag{6}$$

where $K_{\mathrm{I,II}}$ are the modes I and II stress intensity factors, $\sigma(x)$ and $\tau(x)$ are the applied modes I and II crack surface tractions, σ_o and τ_o are the modes I and II cohesive zone stresses, ρ is the plastic zone size, E is the elastic modulus, $m_{\mathrm{I,II}}(x, a)$ are the modes I and II weight functions for the cracked geometry, a is the half crack length, ξ is the crack length dummy variable, and x is the location along the crack surface.

By forcing the modes I and II stress intensity factors at the crack tip to zero, the plastic zone ρ and cohesive zone stresses σ_o and τ_o are found by simultaneously solving the following system

$$\int_0^{a+\rho} \sigma(x)m_{\mathrm{I}}(x,a+\rho)dx - \int_a^{a+\rho} \sigma_o m_{\mathrm{II}}(x,a+\rho)dx = 0 \tag{7}$$

$$\int_0^{a+\rho} \tau(x)m_{\mathrm{II}}(x,a+\rho)dx - \int_a^{a+\rho} \tau_o m_{\mathrm{II}}(x,a+\rho)dx = 0 \tag{8}$$

$$Y^2 = \sigma_o^2 + 3\tau_o^2 \tag{9}$$

where Y is the material yield stress, and $m_{\mathrm{I}}(x, a+\rho)$ and $m_{\mathrm{II}}(x, a+\rho)$ are the modes I and II weight functions for the geometry of interest evaluated at the effective crack length $a+\rho$. Eq. (9) represents the Von Mises yield condition. A yield condition is needed as it provides a relationship between the modes I and II cohesive zone stresses.

A variety of methods could be used to solve Eqs. (7)–(9). For this work, an iterative solution approach was chosen and is discussed in Appendix. To give an example of solutions to Eqs. (7)–(9), the case of constant mixed mode tractions is investigated. Fig. 2a and b presents normalized modes I and II cohesive zone stresses as a function of normalized mode I applied stress for different levels of normalized mode II applied stress. Fig. 3 presents the normalized plastic zone vs. the normalized mode I applied stress for different levels of normalized mode II applied stress.

3. Finite element model

Finite element analysis of a center-cracked plate under plane stress was conducted for the geometry shown in Fig. 4 using ANSYS 11.0 [26]. To approximate an infinite plate, $a/W = 0.1$. To ensure $a/W = 0.1$ is an acceptable approximation, a larger plate with dimensions $a/W = 0.083$ was also investigated. The plate is meshed using four node elements and loaded using modes I and II tractions on the crack face. A bilinear isotropic material model with Young's modulus $E = 70$ GPa, tangent modulus of 700 MPa (0.01E), and yield strength of 350 MPa was used to approximate elastic-perfectly plastic material behavior. A convergence study was conducted to determine an acceptable element size for the analysis. The displacement