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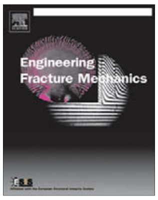

Effects of magneto-electric loadings and piezomagnetic/piezoelectric stiffening on multiferroic interface fracture

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ABSTRACT

Interface fracture of a multiferroic composite is studied by the methods of integral transform and singular integral equation. Parametric studies on the stress intensity factor yield three conclusions. (a) The multiferroic composite is more likely to fracture in electric field than in magnetic field. (b) Under magnetostriction, piezomagnetic stiffening does not affect the interface crack, but the influence of piezoelectric stiffening is notable. Under electrostriction, inverse results are obtained. (c) In magnetic loading cases, the piezomagnetic layer should be softer and the piezoelectric strip stiffer; however, if electric loading is applied instead, opposite conclusion should be expected.

1. Introduction

Due to the intrinsic brittleness of piezomagnetic/piezoelectric materials, the fracture problems of magneto–electro-elastic composites have absorbed the attention of many researchers. In the past years, the crack-surface boundary conditions and the magneto–electro-elastic fracture criteria have been the most extensively and intensively studied topics in this field. However, for fracture analyses on practical smart devices, apart from these two topics, the effects of loading conditions still remain as another significant issue deserving consideration. In fact, different loading conditions might give rise to different fracture behaviors in the same smart device. For example, it is indicated that the loading positions of magneto–electro-mechanical impacts have strong effects on the overshoots of dynamic fracture parameters [1,2].

Smart composites consisting of piezomagnetic and piezoelectric phases are often called multiferroic materials [3,4]. Investigations indicated that the magnetoelectric coupling effects of layered [5] or fiber reinforced [6] multiferroic materials are much larger than those of sintered bulk composites. Therefore, many investigations have been devoted to the former in recent years [7–9]. Practical multiferroic composite devices might serve in either magnetic field or electric field. When they are loaded by magnetic field, deformation may be caused by magnetostriction; if electric loading is applied, electrostriction will induce deformation instead. Magneto/electrostriction, combined with possible mechanical constraints, might further affect the fracture behavior of multiferroic composites.

The deformation in layered multiferroic composites may be different under the action of different loadings. For example, when a layered multiferroic composite is loaded by magnetic field, the piezomagnetic layers would deform initiatively, but the piezoelectric layers would deform passively. That is to say, when loaded by magnetic field, the piezoelectric layers actually act as constraints to the piezomagnetic layers. Similarly, when the composite is loaded by electric field, the latter would

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Nomenclature

w, φ and ϕ	mechanical displacement, magnetic potential and electric potential
τ, B and D	stress, magnetic induction and electric displacement
γ, H and E	strain, magnetic field and electric field
c_{44}, h_{15}, μ_{11}, e_{15} and ε_{11}	elastic constant, piezomagnetic coefficient, magnetic permeability, piezoelectric coefficient and dielectric coefficient
h_1 and h_2	thickness of piezomagnetic layer and piezoelectric strip
a	half-length of crack
τ_0, H_0 and E_0	constraining traction, applied magnetic field and applied electric field
K_e and k_m	piezoelectric stiffening factor and piezomagnetic stiffening factor
$A_w^{(j)}(s)$, $C_w^{(j)}(s)$ $(j = 1, 2)$, $A_\varphi(\xi)$, $C_\varphi(\xi)$, $A_\phi(\xi)$ and $C_\phi(\xi)$	unknown coefficient functions
$g(x)$	dislocation density function
$Q_1(\xi)$ and $Q_2(\xi)$	known functions
q_1 and q_2	known dimensionless constants
$\phi(\tilde{s})$	unknown dimensionless function
$\tilde{K}(a)$	normalized stress intensity factor

serve as constraints to the former instead. Such deformation characteristics might also have some influences on the fracture behavior of layered multiferroic composites. Up till now, the effects of magneto-electric loading conditions on the cracking of multiferroic composites have been scarcely investigated.

In addition, as is well known, piezomagneticity/piezoelectricity may lead to equivalent stiffening. Under the abovementioned initiative or passive deformation, how would the piezomagnetic/piezoelectric stiffening affect the fracture behavior of multiferroic composites? It is also an interesting problem deserving investigation.

Generally speaking, the fracture problem of a smart structure under the action of remote loadings could be viewed as the superposition of two sub-problems. The first is the magneto–electro-elastic problem of an un-cracked structure under remote loadings and the second is the fracture problem of a cracked structure applied by the crack-surface equivalent loadings. Unfortunately, the researchers, in most cases, only pay attention to the second sub-problem. They always simply investigate the fracture responses induced by hypothetic loadings on crack surfaces [10,11]. Though this simple treatment of loadings is sufficient for many fracture problems (e.g., the abovementioned two topics), it is not for all. Because the practical loading conditions are only specifically formulated in the first sub-problem, if we do not solve it we would lose the possibility to study the effects of practical loading conditions.

In the current work, the interface fracture model of a bi-layered multiferroic composite is established. Both sub-problems derived from superposition are solved together. The mixed boundary value conditions of the interface crack are reduced into a singular integral equation, from which numerical results of the stress intensity factor are obtained. Parametric studies are performed and the influences of magneto-electric loadings and piezomagnetic/piezoelectric stiffening are thereby discussed.

2. Problem formulation

Fig. 1 shows the model of a laminated multiferroic composite with a piezomagnetic layer bonded to a piezoelectric strip. The thickness of the former is h_1 and that of the latter is h_2. A crack with half-length a is assumed to be on the interface. The rectangular coordinate system is established with the x axis along the bottom and the y axis through the crack center.

In engineering, the multiferroic composite illustrated by Fig. 1 may be poled along the z axis or y axis, depending on its application purpose. If we want to control or test the in-plane deformation via in-plane magneto-electric fields, the compos-

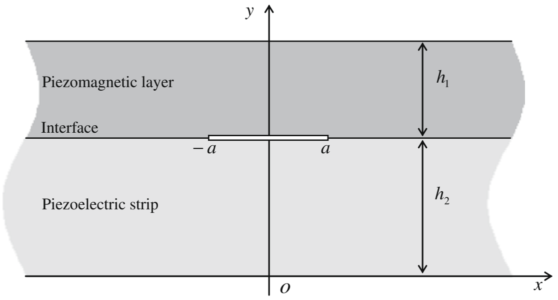

Fig. 1. An interface crack in a bi-layered multiferroic composite.