International Journal of Fatigue 32 (2010) 297–301

Contents lists available at ScienceDirect

International Journal of Fatigue

journal homepage: www.elsevier.com/locate/ijfatigue

Surface crack shape evolution modelling using an RMS SIF approach

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ARTICLE INFO

Article history:
Received 11 June 2008
Received in revised form 28 May 2009
Accepted 11 June 2009
Available online 17 June 2009

Keywords:
Stress intensity factor
RMS SIF
Crack shape
Crack aspect ratio
Paris Law
Surface crack

ABSTRACT

The majority of fatigue cracks in thick plate and tubular sections in structural components are two-dimensional surface cracks having significant propagation lives before becoming critical. The modelling of surface crack propagation life is important across a range of industries from power generation to offshore so that inspection, maintenance and repair strategies can be developed. Linear elastic fracture mechanics based predictions are commonplace, however, unlike thin sections with associated one-dimensional cracks frequently encountered in aerospace industries, crack shape or aspect ratio has a profound effect on crack front stress intensity factor and any resulting Paris Law based life prediction. The two most commonly used approaches are to calculate the crack growth rate at a number of points around the crack front and to consider only surface and deepest points, calculating the relative crack growth rates. Experience using these approaches has shown that the Paris Law coefficient as determined from plane stress specimens appears to lead to previously unpredicted inaccuracies. While this may suggest that the Paris Law is not suitable for this type of cracks, it is believed that a modification in the Paris Law would alleviate this problem. This paper examines this apparent anomaly, explaining why this discrepancy exists and suggests a practical solution using an RMS SIF approach for surface cracks that negates the need to correct the plane stress Paris Law constants.

1. Introduction

Surface cracks account for the majority of structural fatigue failures. Cracks usually initiate from surface defects, which then develop into a part-through crack. Several observations have shown that these cracks are usually semi-elliptical in shape and that in flat specimens these cracks tend to retain a semi-elliptical shape during their growth [1,2].

For the analysis of fatigue crack growth, the Paris Law [3] has proved to be a simple, accurate and robust approach where knowledge of stress intensity factor is enough to predict the growth rate of edge and through-thickness cracks. In order to analyse the growth of surface cracks under cyclic loading, it has been generally accepted to apply the Paris Law for the deepest and the surface points of the crack and then assume a semi-elliptical shape for the crack [2,4]. This has been expressed mathematically in terms of the surface and the deepest points:

$$\frac{da}{dN} = C_{DP}(\Delta K_{DP})^m; \quad \frac{dc}{dN} = C_{SP}(\Delta K_{SP})^m \tag{1}$$

where C_{DP}, C_{SP} and m are material properties and should not depend on the loading and on the geometry of the crack-component configuration.

However, as early as the 1970s it was observed that the two material constants C_{DP} and C_{SP} are not equal. Corn [5] observed that small semi-circular surface cracks under either tensile or bending loads tend to retain their semi-circular shape. Yen and Pendleberry [6] suggested that assuming $C_{SP} = 0.9^m C_{DP}$ can explain this behaviour since the SIF for the deepest point of the small semi-circular crack is about 10% lower than the SIF at the surface point.

Wu [7] argued that in a surface crack, if for each point on the crack front Paris Law is applied as:

$$\frac{da(\phi)}{dN} = C(\phi)[\Delta K(\phi)]^m$$

Then by adopting Corn's observation [5], the following relation can be obtained for $C(\phi)$:

$$C(\phi) = \frac{C_{DP}}{\left[1 + 0.1 \times (1 - \sin\phi)^2\right]^m} \tag{2}$$

In Eq. (2), ϕ denotes the parametric angle on the crack front and C_{DP} corresponds to $C(\frac{\pi}{2})$.

Notwithstanding the fact that Eq. (2) was derived for a crack where $a = c$, i.e. a semi-circular crack, Wu used this formula to

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doi:10.1016/j.ijfatigue.2009.06.012

Nomenclature

a	surface crack depth
c	surface crack half length
C	Paris Law coefficient
K	stress intensity factor (SIF)
$\overline{K}_x$	averaged weight function in the 'x' direction
$\overline{K}_y$	averaged weight function in the 'y' direction
m	Paris Law coefficient
N	number of load cycles
RMS	root mean square
ϕ	surface crack parametric angle

predict crack shape evolution trends for different semi-elliptical surface cracks under bending and tension [7].

In order to take the above analysis one step further, the general problem of the growth of a semi-elliptical surface crack under an arbitrary loading mode can be analysed. Instead of assuming that semi-circular cracks remain semi-circular, it is assumed that semi-elliptical cracks generally tend to retain their semi-elliptical shape. The results of this general analysis confirm the fact that not only is $C(\phi)$ not constant and varies depending on ϕ, but also that it is dependent on crack geometry and loading mode. The load-dependent nature of this coefficient makes its application questionable. Details of this analysis are given in Appendix A. Modification of the ΔK may be an alternative approach to overcome the problem of a variable, load-dependent $C(\phi)$, see the work of Bowness and Lee [8] and Etube et al. [9]. However, utilisation of a modified ΔK would demand correcting the SIF for crack geometry and defining a new method for SIF derivation as a function of σ to satisfy the requirements of a constant C. This would mean that the SIF values obtained using conventional tools such as FE, or other methods such as the rapid calculation method of Albrecht and Yamada [10], would not be applicable for the Paris Law.

Qualitatively, the change in Paris Law coefficient along the crack front can be related to change in the stress state at the crack tip, from a state of high triaxiality of stress at the deepest point, to a biaxial stress state at the surface points. Also, there is a larger plastic region at the crack tip on the surface of the specimen, which further proves the requirement for separate local treatment of the Paris Law in the vicinity of the region.

The analysis given in Appendix A suggests that if the Paris Law is applied to individual points on the crack front in surface cracks, then the coefficients are not merely material properties. It has been shown that $C(\phi)$ generally depends on the geometric parameters of the surface crack, and may also depend on loading. Therefore it has been mathematically shown that the multi-point analysis of surface cracks using Paris Law is not a reliable method of fatigue life estimation. The errors caused from this approach vary depending on the particular problem and in some cases can lead to non-conservative estimates.

Previous authors [11,12] have successfully demonstrated the advantage of the RMS SIF when used for crack growth modelling under certain loadings. However these comparisons are usually presented in a tabular way in which a graphic representation of $\frac{da}{dN}$ vs. ΔK is not shown. The authors believe that this representation is a strong tool in demonstrating the usefulness of the RMS SIF Paris Law. Therefore the results of the current tests should be considered alongside the tests carried out by previous authors [11,12].

2. RMS stress intensity factors

Cruse and Besuner [13] were the first to utilise the concept of an integrated average of the stress intensity factor in what is now known as the root mean square (RMS) stress intensity factor (SIF). RMS SIF is defined, for the two principal growth dimensions, as:

$$\overline{K}_x^2 = \frac{1}{\Delta A_x}\int\int_{\Delta A_x} K^2(s)dA \quad \text{and} \quad \overline{K}_y^2 = \frac{1}{\Delta A_y}\int\int_{\Delta A_y} K^2(s)dA \tag{3}$$

where

$$\Delta A_x = \pi a_y \Delta a_x \quad \text{and} \quad \Delta A_y = \pi a_x \Delta a_y$$

Their method involves definition of a number of characteristic dimensions (usually two) for a crack; the crack propagation being described by keeping track of these dimensions. For the crack shown in Fig. 1, these parameters are a_x and a_y which denote crack lengths in the two perpendicular dimensions, as shown. Cruse and Besuner [13] assumed that the coefficients of the Paris Law for this type of analysis are the same as for when normal stress intensity factor values (i.e. K) are used.

Hence the Paris Law for surface crack growth can be written as:

$$\frac{da}{dN} = C_A(\Delta K_{\mathrm{RMS},A})^m \quad \text{and} \quad \frac{dc}{dN} = C_B(\Delta K_{\mathrm{RMS},B})^m \tag{4}$$

where it is postulated that $C_A = C_B$. If this relation is valid, then the problem of surface crack growth has been immensely simplified. By having two material properties (C and m, which are independent of geometry and loading), it is only sufficient to know the average stress intensity factor values for two directions to predict the crack growth in a specific material.

Before detailing an experimental based examination of the abovementioned assumption, a few points are worth mentioning:

(1) The surface point SIF value imposes a problem since at this point there is an immediate transition in the state of stress from three-dimensional to biaxial. This area usually encompasses a larger plastic zone as a result of this stress state, which means that this single point does not represent the overall crack behaviour near the surface. Another suggested explanation is the different crack closure along the contour of the crack [14,15]. In their experiments, Kim and Song [15] observed that the crack opening ratio was about 10% greater at the deepest point than at the surface.
(2) It was shown that $C(\phi)$ varies and is a function of crack geometry and loading. An averaged stress intensity factor, when used with the newly proposed form of the Paris Law, would essentially eliminate these variations.
(3) This new method intrinsically ensures that cracks retain their semi-elliptical shape, and no extra geometrical considerations are required.

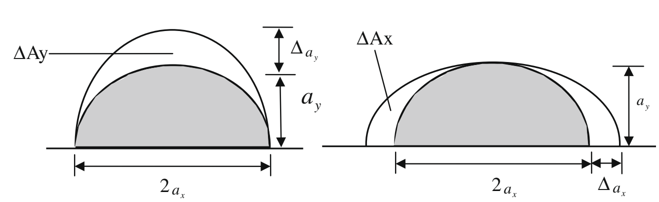

Fig. 1. Two characteristic growth dimensions.