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Crack growth-based fatigue-life prediction using an equivalent initial flaw model. Part II: Multiaxial loading

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ABSTRACT

A general methodology is proposed in this paper for fatigue-life prediction using crack growth analysis. This is the part II of the paper and focuses on the fatigue-life prediction under proportional and nonproportional multiaxial loading. The proposed multiaxial fatigue-life prediction is based on a critical plane-based multiaxial fatigue damage model and the Equivalent Initial Flaw Size (EIFS) concept. An equivalent stress intensity factor under general multiaxial proportional and nonproportional loading is defined. The fatigue life is predicted by integration of the crack growth rate curve from the EIFS to the critical crack length. The proposed model can automatically adapt for different materials experiencing different local failure modes. The numerical fatigue-life prediction results calculated by the proposed approach are validated with experimental data for a wide range of metallic materials available in the literature. Reasonable agreements are observed between the model predictions and the experimental observations under proportional and nonproportional loading.

1. Introduction

Many mechanical and structural components experience multiaxial cyclic loadings in service, e.g. the mast in a helicopter, railroad wheels, turbine blades, drive shafts, etc. [1–3]. Anisotropy of materials can also cause multiaxial fatigue problem even under the uniaxial loading, e.g. multidirectional composite laminate [4]. The multiaxial fatigue problem is more difficult due to its complex stress states, nonproportional loading histories and various initial crack orientations [5]. Although extensive efforts have been made in the past decades there is no universally accepted model available. Several reviews and comparisons of existing multiaxial fatigue models can be found in [6–10].

Fatigue-life prediction can be generally classified into two approaches: the stress (strain)-life approach and the fracture mechanics-based approach. Most existing multiaxial fatigue theories were developed based on the stress (strain)-life approach. A brief review is given below.

The stress based approaches can be classified into four categories: empirical equivalent stress model, stress invariants model, average stress model, and critical plane-based model [5]. Gough and Pollard [11,12] suggested two empirical equivalent stresses for multiaxial fatigue analysis of metals under combined proportional bending and torsion. Their proposed criteria does not address nonproportional loading. Lee [13] presented an empirical design criterion for fully reversed nonproportional torsion and bending by modifying Gough's ellipse quadrant [11]. The drawback of Lee's criterion is that many experimental data is required for model calibration. Sines [14] developed a high-cycle fatigue criterion using the mean values of the shear and normal stresses. This model was only used for ductile materials under the fatigue limit regime.

Various models based on stress invariants were proposed in [15–20]. Sines [15] used the stress invariants for high-cycle fatigue analysis and introduced a linear dependence of the bending limit upon a superimposed static normal stress. In his proposed model [15], the ratio of fatigue limits in torsion and in fully reversed bending remains constant for all metals, which is not supported by experimental results. Crossland [17] suggested a similar criterion as the Sines' model [15], but considered the influence of the hydrostatic stress. The uniqueness of the torsion fatigue limit is correctly reproduced. Kakuno and Kawada [19] proposed a design formula by separating the effects of the amplitude and the mean value of the hydrostatic stress. The proposed method [19] is not applicable for all nonproportional loading conditions. One major limitation of the stress invariant approach is that it cannot predict the orientation of the initiated fatigue crack [7], which is another important characteristic of the multiaxial fatigue problem.

Another approach is the average stress approach, which uses the average stress within an area/volume as the damage indicator. Papadopoulos et al. [7] proposed an average stress approach using the average value of the stress components involving the critical point. This model is limited to hard metals for which the ratio of

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t_{-1}/f_{-1} (the fully reversed torsion fatigue limit over the fully reversed bending fatigue limit) is between $1/\sqrt{3}$ and 0.8. Another limitation of the model is that nonproportional loading has no effect on the life prediction which is not consistent with experimental observations [6]. Sonsino and Crubisic [21] observed that the decrease of fatigue life under out-of-phase strains was caused by of the change of principal strain directions, which results in an interaction of the deformations in all directions on the surface. This interaction could be accounted for by the arithmetic mean of the shear amplitudes acting in all interference planes on the surface. Sonsino and Crubisic's method is originally proposed for sinusoidal loading only. Liu and Zenner [22] also introduced a criterion based on the space averages to explain the multiaxial fatigue behavior.

In the past decades, fatigue-life prediction criteria based on the critical plane approach became widely used because they generally predicted the fatigue damage more accurately [23]. The critical plane approach is based on the physical observations that fatigue cracks initiate and grow along certain planes in the material. This concept was firstly proposed by Stanfield [24], and has been developed since then by other researchers [25]. Various critical plane-based models that use the *S–N* (e–N) curves have been proposed. Findley [26] and Matake [27] presented a similar criterion for high-cycle multiaxial fatigue analysis using the shear stress amplitude and the maximum value of the normal stress on the critical plane. Findley [26] determined the critical plane by maximize a linear combination of the shear stress amplitude and the maximum value of the normal stress. Matake [27] defined the critical plane as the one experiencing the maximum shear stress amplitude. McDoarmid [28] used the concept of case A and case B cracks introduced by Brown and Miller [29] and proposed a generalized failure criterion that takes the crack initiation modes into consideration. However, this criterion is limited to the range of loading conditions and does not explain the mean stress effect. Fatemi and Socie [30] modified the parameter in the Brown and Miller's approach [29] to account for the additional cyclic hardening during nonproportional loading. Carpinteri and Spagnoli [8,31,32] proposed that the critical plane orientation is determined by the principal stress directions through a weight average function under nonproportional loading. Liu and Mahadevan [5] proposed a unified multiaxial fatigue damage model based on the critical plane approach. One unique property of the proposed model is that the critical plane is related to material ductility and varies for different local failure modes. The applicability of the proposed model [5] is significantly improved.

Multiaxial fatigue models based on the *S–N* curve approach are not suitable for damage tolerance analysis, which is based on the fracture mechanics. In this paper, a multiaxial fatigue life model is proposed based on the crack growth analysis. The proposed methodology integrates a previously developed multiaxial fatigue model [33] and a general life prediction methodology based on the Equivalent Initial Flaw Size (EIFS) concept [34]. The proposed multiaxial model is applicable to a wide range of ductile and brittle metals. It does not require solving the inverse crack growth problem, which makes the computation very efficient. A wide range of experimental data for different metallic materials is used for model validation.

A similar method was proposed by Döring et al. [35], i.e. the short crack model is based on the critical plane concept and a starter crack length a_0 is used for the fatigue-life prediction. The starter crack length in [35] is determined by backward integration of the Paris type crack growth equation. The proposed EIFS concept is different from the commonly used backward integration and is easy to be calculated, i.e. no iterative calculation is required. Also, it is independent of applied load level, which is one of common drawbacks of the backward integration method [34]. As for the critical plane method, the one used in [35] is based on the critical plane concept proposed by Brown and Miller [29] and Fatemi and Socie [30]. The fatigue damage is accumulated in the same way for different materials under the same stress state and their applicability generally depends on the material's properties. In our paper, the critical plane depends on both the stress state and the material properties. One of the advantages of the proposed critical plane method is that it can automatically adapt for different materials experiencing different local failure modes. Detailed comparison of the proposed critical plane method and other critical plane method can be found in [5,36].

2. Proposed methodology

2.1. Mixed mode fatigue crack growth

The critical plane-based model for multiaxial fatigue damage analysis proposed by Liu and Mahadevan [33] is summarized below. Detailed derivation and validation can be found in the referred article. The general fatigue limit criterion under multiaxial loading is expressed as

$$\sqrt{\left(\frac{\sigma_c}{f_{-1}}\right)+\left(\frac{\tau_c}{t_{-1}}\right)+A\left(\frac{\sigma^H}{f_{-1}}\right)}=B \tag{1}$$

where σ_c and τ_c are the normal stress range and shear stress range acting on the critical plane for both nonproportional loading and proportional loading, respectively. σ_H is the hydrostatics stress amplitude. A and B are the material parameters which can be determined from uniaxial and torsional fatigue limits. Material parameters A, B, and γ are listed in Table 1. The material parameter $s=t_{-1}/f_{-1}$ is related to the material ductility and affects the critical plane orientation. In Eq. (1), the ranges of the stress components σ_c and τ_c are evaluated by taking the difference of the maximum value and the minimum value on the critical plane. For general nonproportional loading case, this is done by enumeration. Details can be found in [35] using the proposed critical plane method. In this paper, only the constant proportional and nonproportional loading is considered. Therefore, the ranges of the stress components are calculated using the maximum and minimum value during one external loading cycle.

For brittle materials, the critical plane is close to the maximum normal stress amplitude plane. For ductile materials, the critical plane is close to the maximum shear stress amplitude plane. Thus, this model can automatically adapt for different failure modes, i.e. tensile or shear dominated failures [36]. The critical plane is load path-dependent since different loading paths result in different maximum normal stress plane. Therefore, the proposed model includes the loading path effect and the nonproportional loading influence on the fatigue life, which has been discussed in detail in [33].

Kitagawa diagram [37] and El Haddad's model [38] is used to link the multiaxial fatigue limit criteria to the fatigue crack growth threshold stress intensity factor. The fatigue limit can be expressed using the threshold stress intensity factor and a fictional crack length a [33] as

$$f_{-1}=\frac{K_{I,th}}{\sqrt{\pi a}} \tag{2}$$

Table 1
Material parameters for fatigue limit criterion.

Material property	$s=\frac{t_{-1}}{f_{-1}}\leqslant 1$	$s=\frac{t_{-1}}{f_{-1}}>1$
γ	$\cos(2\gamma)=\frac{-2+\sqrt{4-4(1/s^2-3)(5-1/s^2-4s^2)}}{2(5-1/s^2-4s^2)}\leqslant 1$	$\gamma=0$
A	$A=0$	$A=9(s^2-1)$
B	$B=[\cos^2(2\gamma)s^2+\sin^2(2\gamma)]^{\frac{1}{2}}$	$B=s$