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Remarks on multiaxial fatigue limit criteria based on prismatic hulls and ellipsoids

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ABSTRACT

This paper focuses on a class of multiaxial fatigue limit criteria where the equivalent shear stress amplitude is calculated by means of a scalar measure associated with a hypersurface enclosing the deviatoric stress history at a material point. We consider two hypersurfaces proposed by the authors, namely the maximum prismatic hull and the minimum Frobenius norm ellipsoid. Previous results obtained with elliptic and non-elliptic stress paths strongly suggested that such measures might always be the same. In this work we consider two counter-examples which show that these approaches are distinct. Fatigue limit criteria based on the linear combination of these measures with the maximum hydrostatic stress were applied to experimental data including: axial–torsional, biaxial tension and plane stress tests performed under harmonic and non-harmonic, synchronous and asynchronous waveforms. The predictions for both criteria fell within a 15% scatter band.

1. Introduction

High-cycle fatigue is a failure mode which can occur at a material point of a mechanical component due to variations of macroscopic elastic stresses which activate damage at the mesoscopic scale [1–6]. In this multiscale framework, the fatigue process may lead to one of the following situations: whenever the accumulated mesoscopic damage is unbounded, the initiation of a visible crack is expected and the component will eventually fail after a finite period of solicitation; on the other hand, when the accumulated damage is bounded, a condition termed mesoscopic shakedown, the component is presumed to indefinitely (in practice more than 10^6 cycles) sustain the applied loading.

We consider the formulation and assessment of fatigue limit criteria based on new definitions for the shear stress amplitude within the setting of multiaxial stress histories. In particular, we assess the following measures proposed by the present authors: the maximum prismatic hull [7,8] and the minimum Frobenius norm (F-norm) ellipsoid [6]. Roughly speaking, in [7,8] the shear stress amplitude is associated with the sizes of the faces of a properly oriented rectangular prismatic hull enclosing the stress history, whereas [6] considers the radii of the minimum ellipsoid enclosing the stress history.

Our motivation for this work is that previous investigation [7,8] showed that the application of the maximum prismatic hull and the minimum F-norm ellipsoid to compute the amplitude of elliptic paths are equivalent. Besides, for some non-elliptical paths the computation of these measures also provided the same values [9]. These results strongly suggested that such measures might always be the same. Here we evaluated these measures against a wider range of non-elliptical paths and detected that in two cases the measures are different. In addition to this main result, we assessed fatigue criteria based on these measures against a large set of multiaxial fatigue limit data – including axial–torsional, biaxial tension and plane stress tests performed under harmonic and non-harmonic, synchronous and asynchronous waveforms. The predictions for both criteria fell within a 15% scatter band. For comparative purposes, other studies which analysed some of the experimental data considered in this work can be found in [10–13].

2. Preliminary definitions

The stress state at a material point is denoted by $\boldsymbol{\sigma}$. It may be decomposed into spherical and deviatoric parts as

$$\boldsymbol{\sigma} = \sigma_m \boldsymbol{I} + \boldsymbol{S} \qquad (1)$$

where $\sigma_m := (\mathrm{tr}\,\boldsymbol{\sigma})/3$ is the mean (or hydrostatic) stress, $\mathrm{tr}\,(\cdot)$ is the trace operator and $\boldsymbol{I}$ is the identity tensor. The deviatoric tensor is given by $\boldsymbol{S} = \boldsymbol{\sigma} - \sigma_m \boldsymbol{I}$ and satisfies $\mathrm{tr}\,\boldsymbol{S} = 0$.

We shall consider the following orthonormal basis for the five-dimensional space of symmetric deviators

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$$\mathbf{d}^{(1)} := \frac{1}{\sqrt{6}}(2\mathbf{e}^x \otimes \mathbf{e}^x - \mathbf{e}^y \otimes \mathbf{e}^y - \mathbf{e}^z \otimes \mathbf{e}^z)$$
$$\mathbf{d}^{(2)} := \frac{1}{\sqrt{2}}(\mathbf{e}^y \otimes \mathbf{e}^y - \mathbf{e}^z \otimes \mathbf{e}^z)$$
$$\mathbf{d}^{(3)} := \frac{1}{\sqrt{2}}(\mathbf{e}^x \otimes \mathbf{e}^y + \mathbf{e}^y \otimes \mathbf{e}^x)$$
$$\mathbf{d}^{(4)} := \frac{1}{\sqrt{2}}(\mathbf{e}^x \otimes \mathbf{e}^z + \mathbf{e}^z \otimes \mathbf{e}^x)$$
$$\mathbf{d}^{(5)} := \frac{1}{\sqrt{2}}(\mathbf{e}^y \otimes \mathbf{e}^z + \mathbf{e}^z \otimes \mathbf{e}^y) \quad (2)$$

where $\mathbf{e}^x$, $\mathbf{e}^y$ and $\mathbf{e}^z$ are unit vectors of a coordinate system and $\otimes$ denotes tensor product. Then, a stress deviator is represented as the linear combination

$$\boldsymbol{S} = \sum_{i=1}^{5} S_i \mathbf{d}^{(i)} \quad (3)$$

with components given as

$$S_1 = \frac{1}{\sqrt{6}}(2\sigma_x - \sigma_y - \sigma_z), \quad S_2 = \frac{1}{\sqrt{2}}(\sigma_y - \sigma_z),$$
$$S_3 = \sqrt{2}\sigma_{xy}, \quad S_4 = \sqrt{2}\sigma_{xz}, \quad S_5 = \sqrt{2}\sigma_{yz} \quad (4)$$

The Frobenius norm of a deviatoric tensor is defined as

$$||\boldsymbol{S}|| = \sqrt{\boldsymbol{S}\cdot\boldsymbol{S}} = \sqrt{S_1^2 + S_2^2 + S_3^2 + S_4^2 + S_5^2} \quad (5)$$

3. Fatigue limit criteria

The input data for a high-cycle fatigue analysis is defined by a set of elastic stresses $\Delta = \{\boldsymbol{\sigma}^k, k = 1 : m\}$ at a material point. This information can be the result of purely elastic or steady-state elasto-plastic responses of a mechanical component subject to a variable loading. In the former case, the applied loading is such that no plastic deformations are produced during the entire life of a mechanical component and hence the input stress data are the result of m elastic analyses. In the latter case, called elastic shakedown, the applied loading produces a bounded amount of plastic deformation leading to a fixed residual stress distribution which renders a purely elastic response. The computation of the elastic shakedown state may be performed either by incremental or direct approaches [14–16].

The fluctuation of macroscopic elastic stresses at a material point can produce material transformations at the mesoscopic level. In this setting, failure by high-cycle fatigue is the consequence of inelastic mesoscopic adaptation, i.e. an unbounded damage process which leads to the initiation of a macroscopic crack. On the other hand, infinite endurance shall take place if the macroscopic stress variations induces mesoscopic elastic shakedown.

In order to construct a mathematical model to quantify the fatigue limit phenomenon, we assume that the stresses Δ render infinite endurance if

$$F(\Delta) \leqslant 0 \quad (6)$$

where the admissibility function $F(\cdot)$ characterizes the fatigue behavior of a material. Following [6–8,17], the fatigue limit criteria investigated in this work are written as

$$F(\Delta) := \frac{1}{\sqrt{2}} S_a + \alpha\sigma_{mc} - \beta \leqslant 0 \quad (7)$$

where $S_a = S_a(\Delta^{\text{dev}})$ is an amplitude associated with the set of deviatoric stresses $\Delta^{\text{dev}} = \{\boldsymbol{S}^k, k = 1 : m\}$, $\sigma_{mc} = \max_{\sigma\in\Delta}\{\sigma_m\}$ is the maximum hydrostatic stress, while α and β are material parameters. Fatigue criteria of this type are insensitive to a superimposed mean shear stress, but sensitive to hydrostatic stresses, as commonly observed in high-cycle fatigue of metals (see [18–20] and references therein).

For this class of fatigue criteria, the main issue lies in a proper definition of the stress amplitude within the setting of a five-dimensional deviatoric space. In order to get a geometrical insight into this problem, it is helpful to observe that, when stresses are proportional or affine, a natural measure would be half the length of the line segment described by the deviatoric stresses. On the other hand, for more complex paths, a number of different amplitudes may be defined (see, for instance, Fig. 1). In order to tackle this problem, one of the approaches proposed in the literature [6–8,21,22] defines a hyper-solid which encloses the deviatoric stresses acting on a material point and then chooses some scalar measure associated to the hyper-solid which quantifies the induced fatigue damage.

Next, we consider some definitions for the deviatoric stress amplitude. The model proposed by Crossland [17] considers that the shear stress amplitude is the radius of a ball enclosing the deviatoric stress history. This quantity can be computed as

$$S_a := \min_{\boldsymbol{A}} \max_{\boldsymbol{S}} \{||\boldsymbol{S} - \boldsymbol{A}|| \ |\boldsymbol{S} \in \Delta^{\text{dev}}\} \quad (8)$$

where $\boldsymbol{A}$ stands for the center of the ball. It should be remarked that this model is not able to distinguish linear and affine paths from more complex ones, although experimental observations have shown that these paths may produce distinct fatigue damage [20,21,23].

Mamiya and Araújo [7,8,24] propose a measure for the shear stress amplitude based on the largest prismatic hull enclosing the deviatoric stress path. Let

$$a_i(\theta) := \frac{1}{2}(\max S_i - \min S_i), \quad i = 1:5 \quad (9)$$

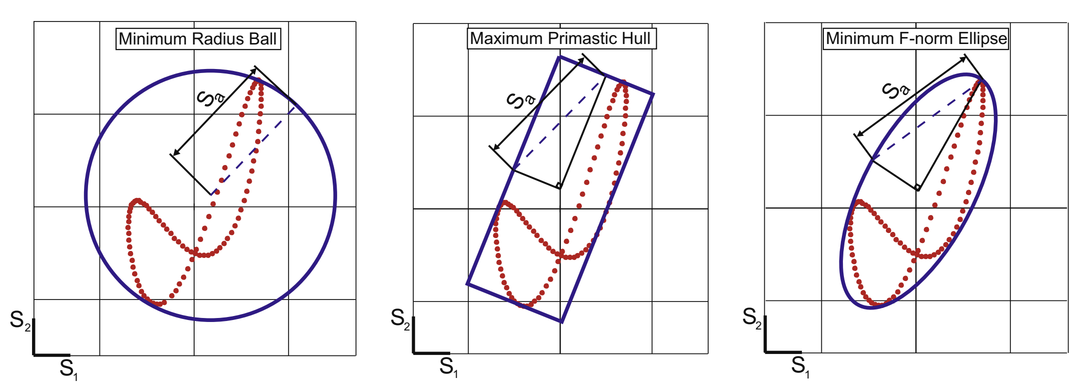

Fig. 1. Definitions for the shear stress amplitude.