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A novel frequency domain method for predicting fatigue crack growth under wide band random loading

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Abstract

This work deals with the evaluation of the fatigue crack growth rate of structural components subjected to uniaxial Gaussian stationary wide band random loading. In detail, a new frequency domain method that allows the user to estimate the expected crack growth rate directly from the PSD data is proposed. Using a stochastic mean function properly, introduced and described by simple closed form relationships implemented by systematic numerical simulations of a high number of wide band random processes, the proposed method permits to avoid the onerous time domain simulations and provides in general crack growth rate predictions in a good accordance with the so-called time domain method. Practical applications, carried out by considering various PSDs reported in the literature, have corroborated the accuracy of the proposed method.

Keywords: Fatigue; Crack growth; Random process; Power spectral density; Random loading; Rainflow method; Stochastic analysis

1. Introduction

Structural components of machines and mechanical systems are often subjected to cyclic loading that can lead to the well known fatigue damage [1,2], consisting in general into the raising and the successive propagation of a fatigue crack. In the fatigue analysis the most relevant feature is the way in which the successive load extrema (maxima and minima) follow each other.

As it is well known, the experimental evidence shows that in the presence of a generic cyclic loading history, the crack growth rate is related mainly to the characteristic of the fatigue cycles, as the range r, given by the difference between the maximum and the successive minimum, and the mean m of the same values.

In particular, for variable amplitude loading history, the fatigue cycles have to be obtained by applying a proper counting method [3], such as the rain-flow algorithm, that can be considered the most accurate method because it allows the user to avoid the underestimation of the damage due to the possible decomposition of a low frequency cycle into many high frequency excursions of limited amplitude [2].

After the fatigue cycles counting, the crack propagation is evaluated by accumulating the crack growth due to each single cycle, evaluated by a proper propagation law (Paris law, Forman law, etc.) [4,5].

Under random loading, as that due to turbulence, wind, road roughness, vibration or marine weaves, in which both the amplitude r and the mean m vary in a random manner, such an approach can be used to evaluate the expected fatigue damage if a sufficient number of experimental tests or numerical simulations, are available to a reliable statistical analysis.

Unfortunately, due to both the long registration/simulation time and the long processing time, this approach is in general quite time consuming; therefore, it cannot be used at the design stage for a simple and rapid fatigue damage prediction.

In turn, modelling the load history as a random stress process [2,6], the fatigue damage prediction can be accomplished

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Nomenclature

a	crack length (semi-length for centred crack)
C,n	material constants involved in crack propagation laws
$E[\bullet]$	stochastic mean operator
F	shape function
$H_X(\tau)$	autocorrelation function of the process $X(t)$
h	stochastic mean function defined by Eq. (27)
M	stochastic mean function defined by Eq. (20)
m	mean value of fatigue cycle
N	number of cycles
$\overline{N}$	expected number of cycles
$N_{X,0}$	expected number of zero-crossing of $X(t)$ per unit time
$N_{\dot{X},0}$	expected number of peaks of $X(t)$ per unit time
$p_{r,m}(r,m)$	joint probability density function of r and m
$p_m(m)$, $p_r(r)$	marginal probability density functions of m, r
R	fatigue ratio
r	range of fatigue cycle
$S_X(\omega)$	power spectral densities of $X(t)$
T	time involved by N fatigue cycles
$\overline{T}$	expected time corresponding to $\overline{N}$ cycles
V	crack growth rate
$\overline{V}$	expected crack growth rate
α_X	irregularity factors of $X(t)$
β_X	wide band parameter of $X(t)$
$\Gamma[\bullet]$	gamma function
γ	non-dimensional residual life parameter defined by Eq. (18)
$X(t)$	random stress process
λ_i	i-order spectral moment of $X(t)$
μ_X	mean value of $X(t)$
ρ_x	radii of gyration of $S_X(\omega)$ around the frequency origin
$\rho_{\dot{x}}$	radii of gyration of the derivative of $S_X(\omega)$around the frequency origin
σ_X	standard deviation of $X(t)$
σ_X^2	variance of $X(t)$
ω	circular frequency (rad/s)
Ω_X	abscissas of the centroid of $S_X(\omega)$

in the frequency domain. In particular, if the random stress process is Gaussian and stationary, then the peaks sequence depends on the distribution of the energy process in the frequency domain, described by the so-called power spectral density (PSD).

The relationship between the PSD and the statistical distribution of the stress extrema (peaks and valleys) has been obtained theoretically for both Gaussian narrow band and Gaussian wide band processes [2,6], whereas the theoretical relationship between the statistical distribution of the fatigue cycles and the PSD is known only for Gaussian narrow band processes. Therefore, at present for Gaussian wide band processes accurate analysis of the fatigue cycles distribution, and consequently of the average crack growth rate, cannot be performed at the design stage also if a proper propagation law is used.

In order to overcome this drawback, several approximate methods have been proposed in the literature to estimate the fatigue damage of structural components subjected to random stress process [7–20].

Some of these methods [7,9] work in the frequency domain (frequency domain methods) and relate directly the fatigue cycle distribution to the characteristics of the PSD; other methods [10–20] work in the time domain (time domain methods) and predict the fatigue cycle distribution or the average crack growth rate by introducing various approximate assumptions.

In the present paper, on the basis of systematic numerical simulations a new frequency domain method that allows the user an accurate evaluation of the average growth rate of a crack located into structural components subjected to a stationary wide band Gaussian random stress process with null mean value, is implemented.

2. Preliminary concepts and definitions

2.1. Random stress processes

A random process $X(t)$ is a random variable that depends on a deterministic parameter t [21]. In the examined case of stress time histories, t is the time. If the random process is stationary and Gaussian, i.e. if its probability density function $p_X(x)$ is Gaussian and independent on time, then in time domain it is univocally characterized by the autocorrelation function $H_X(\tau)$ [21]:

$$H_X(\tau) = E[X(t)X(t+\tau)] - \mu_X^2 \tag{1}$$

being $E[\bullet]$ the stocastic average operator and μ_X the mean of the process. As it is well known, $H_X(\tau)$ is a continuous, real, odd and limited function whose maximum value $H_X(0)$ coincides with the variance σ_X^2 of the process, i.e. $H_X(0) = \sigma_X^2$.

In the frequency domain a stationary Gaussian process is univocally characterized by the power spectral density (PSD) function $S_X(\omega)$, given by the Fourier transform of the autocorrelation function [21]:

$$S_X(\omega) = \int_{-\infty}^{+\infty} \mathrm{e}^{-\mathrm{j}\omega\tau} H_X(\tau)\mathrm{d}\tau \tag{2}$$

Due to the properties of the Fourier transform, $S_X(\omega)$ is an even positive function, whose area is equal to the variance σ_X^2 of the process [21].