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# A viscoelastic-viscoplastic model with non associative plasticity for the modelling of bonded joints at high strain rates

Ludovic Dufour*, Benjamin Bourel, Franck Lauro*, Gregory Haugou, Nicolas Leconte

*University of Valenciennes and Hainaut Cambrésis, LAMIH, UMR CNRS 8201, F-59313 Valenciennes, France*

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ABSTRACT

The purpose of this work is to present a constitutive model able to represent the behaviour of rubber-toughened adhesive joints under dynamic loading. A fully coupled viscoelastic–viscoplastic damage model at finite strains developed for mineral filled semi-crystalline polymer is identified for an epoxy adhesive. The parameters of the model are identified from experimental tests undertaken on bulk material in compression and tension at several loading speeds. In order to validate the accuracy of the model to represent bonded joints, a specific dynamic Arcan device is used. Experimental tests on this apparatus are carried out for tensile, shear and mixed tensile/shear loadings at 1 mm/s, 10 mm/s and 100 mm/s. A very good agreement between experimental and numerical results is obtained for a large strain rate range and for various stress states.



## 1. Introduction

The use of adhesive-bonding is becoming increasingly common in the automotive industry, which aims at improving structural performance and reducing the vehicle weight. Recently, a new generation of adhesives (toughened adhesives) has been developed in order to enhance the ductility of adhesives used in vehicle bodies and to improve their performance under dynamic loadings. The matrix of epoxy-based adhesives in particular is modified by addition of nodules affecting plasticity and other mechanical properties such as viscoelasticity, viscoplasticity and damage evolution during plasticity [8,15]. This leads to more complex adhesives, which increases the difficulties of modelling, particularly, for vehicle crashworthiness analysis.

Various methods are used to model the adhesive joint. The first one is based on a discrete approach which consists in using cohesive elements [19,26]. The second one is a continuous approach based on the use of standard finite elements with a fine description of the constitutive law. An overview of adhesive-bond modelling in crash simulations with a comparison between the continuum elements and the cohesive ones is given in [10].

With the continuum approach, the stress-strain relation for adhesive materials shows similar characteristics as that observed in the inelastic behaviour of polymers. The elasto-plastic (or visco-plastic) material models based on continuum theories or micro-mechanisms are generally used. Classical elasto-plastic models with pressure-independent yield conditions (eg. von Mises) are widely available in commercial codes and widely used since they require a low number of input parameters [25]. However, these criteria cannot accurately predict the behaviour of adhesives under multiaxial loading, as yielding in these materials is sensitive to hydrostatic as well as deviatoric stresses [22]. As a consequence, pressure-dependent plasticity theory is more appropriate in this case. In this context, the Drucker-Prager yield criterion is widely used for polymeric materials [18]. In addition, the mechanical properties of polymers, including structural adhesives, are generally sensitive to the strain rate [4,15]. The effect of the strain rate sensitivity on the behaviour of bulk materials of epoxy resin adhesives is analysed in [14]. It leads to complex behaviours which could be described by different models as non-linear viscoelasticity [1], viscoplasticity [21], coupled elasto-viscoplasticity [9,16], viscoelasticity–viscoplasticity [11].

Two kinds of tests are generally used for the mechanical characterisation of adhesive. The first kind is achieved on bulk adhesive specimens [15,18]. It leads to a direct identification of the stress/strain relation. These tests are only limited by the difficulty to obtain pore-free samples due to the original packaging of the adhesive. The strain-rate effect can be determined by using digital image correlation technique (DIC) [17]. The second kind of test is achieved on assemblies. It permits a characterisation of bonded joints in conditions close to those used in industry, with a thin adhesive layer. But these experiments suffer from a global response highly sensitive to the adherent properties. Moreover, high stress

* Corresponding authors.
*E-mail addresses:* ludovic.dufour@univ-valenciennes.fr (L. Dufour), franck.lauro@univ-valenciennes.fr (F. Lauro).

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heterogeneities near the edge of the joint do not allow to directly obtain the constitutive law and a reverse identification is often necessary. Some works deal with the reduction of the edge effect by the use of a special Arcan testing device in quasi-static [5] and in dynamic [7] loading conditions. The final objective of all these works is to determine the failure occurrence of the joint. The knowledge of the stress state into the joint is then mandatory. The model generally used or experiments often limit this knowledge.

The objective of this paper is to focus on the identification and modelling of the SikaPower498 adhesive behaviour under multiaxial loadings for a large strain rate range (0.1 $s^{-1}$ up to 500 $s^{-1}$). To reach this objective, specific dynamic tests have been performed, first, on bulk specimen associated to an original approach with digital image correlation analysis, and secondly, with a new Arcan device especially designed to reach high strain rates into the joint.

Concerning the modelling, the complex behaviour of the adhesive is taken into account by using a viscoelastic–viscoplastic model which is an extension of the elasto-viscoplastic model proposed for polymer by Balieu [2,3].

## 2. Constitutive model description

### 2.1. Viscoelasticity

The viscoelastic model is coupled with a viscoplastic model to take the strain rate sensitivity into account at an early stage of the deformation process. The behaviour of the material is based on the assumption that the rate of deformation $D^t$ is divided into viscoelastic and viscoplastic parts such as :

$$D^t = D^{ve} + D^{vp} \tag{1}$$

This decomposition of the rate of deformation is linked to the hypoelastic formulation which requires for large deformations to achieve incremental objectivity in order to ensure material frame indifference during large deformations/rotations [2]. The linear Wiechert viscoelastic model is used in the constitutive model in order to represent the linear strain rate dependency and the multitude of relaxation times. By combining the n Maxwell elements with the Hooke element introduced in the standard linear solid model to obtain a long term stress response, it results the linear viscoelastic Wiechert model (i.e. generalised Maxwell model) illustrated in Fig. 1.

In the Wiechert model, the strain on each element is the same as the total strain, and the total stress is the sum of the individual stresses. Applying the superposition theorem, the expression of the stress at time $t$ for the Wiechert model is given by:

$$\sigma^{ve}(t) = \left[E_\infty + \sum_{i=1}^{n} E_i^{ve} exp\left(-\frac{t}{\tau_i}\right)\right]\varepsilon_0 \tag{2}$$

where $E_\infty$ is the added Hooke element stiffness, $E_i$ and $\tau_i$ are the Young modulus and the relaxation time of the $i$th Maxwell element, respectively.

To extend this model in the three dimensions and to take the history of the deformation at time $t$ into account, a Boltzmann superposition principle (or integral) is applied by summing stress increments due to the deformation increment $d\varepsilon(\tau)$ at the previous time $\tau$. This leads to a linear relation between the stress and the strain rate:

$$\sigma^{ve}(t) = \int_{-\infty}^{t} R^{ve}(t-\tau) : \frac{d\varepsilon(\tau)}{d\tau} d\tau \tag{3}$$

where $R^{ve}$ is the fourth order relaxation tensor expressed in terms of Prony series as:

$$R^{ve}(t) = \mathscr{L}_\infty^{ve} + \sum_{i=1}^{N} \mathscr{L}_i^{ve} exp\left(\frac{-t}{\tau_i}\right) \tag{4}$$

$\mathscr{L}_\infty^{ve}$ is the fourth order long term elastic tensor and $\mathscr{L}_i^{ve}$ the fourth order elastic stiffness tensor of the $i$th Hooke element, defined by:

$$\mathscr{L}_\infty^{ve} = 2G_\infty I_d + K_\infty I \otimes I, \; \mathscr{L}_i^{ve} = 2\,G_i I_d + K_i I \otimes I, \tag{5}$$

with $I_d$ the deviatoric projection tensor such as:

$$I_d = I_s - \frac{1}{3} I \otimes I \tag{6}$$

and with $I_s$ and $I$ the fourth order symmetric identity and second order identity tensors. As in classical elasticity, the bulk and shear long term moduli are defined by:

$$G_\infty = \frac{E_\infty}{2(1+\nu)}, K_\infty = \frac{E_\infty}{3(1-2\nu)} \tag{7}$$

### 2.2. Viscoplasticity

The commonly used viscoplastic approach is the von Mises plasticity model. However, most polymers have a different behaviour in tension, compression, and shear, and present a volume variation in plasticity so the von Mises yield surface is not correct. To take these aspects into account a non-associative flow rule and a pressure dependent criterion need to be assumed. A non symmetrical yield surface is then used to represent the different behaviours under tension, compression, and shear. This yield surface initially proposed by Raghava [23], expressed in terms of the nominal stress $\tilde{\sigma}$, assumes that the plasticity occurs when the first invariant of the stress tensor ($I_1(\tilde{\sigma})$) and the second invariant of the deviatoric stress tensor ($J_2(\tilde{S})$) reach a critical combination described by:

$$f(\tilde{\sigma}, R) = \frac{(\eta-1)I_1(\tilde{\sigma})\sqrt{(\eta-1)^2 I_1^2(\tilde{\sigma}) + 12\eta J_2(\tilde{S})}}{2\eta} - \sigma_t - R(\kappa) \tag{8}$$

with $\kappa$, the equivalent plastic strain defined by:

$$\kappa = \sqrt{\frac{2}{3}\varepsilon_p : \varepsilon_p} \tag{9}$$

where $J_2(\tilde{S})$ is the second invariant of the effective deviatoric stress tensor:

$$J_2(\tilde{S}) = \frac{1}{2}\tilde{S} : \tilde{S} \tag{10}$$

and $I_1(\tilde{\sigma})$ is the first invariant of the effective stress tensor given by:

$$I_1 = tr(\tilde{\sigma}) \tag{11}$$

Finally the parameter $\eta$ characterizes the hydrostatic pressure dependency. It is obtained by using the ratio between the yield stress in compression and tension. It corresponds to the usual way to obtain this parameter even if the hydrostatic stress states are close [23]:

$$\eta = \frac{\sigma_y^c}{\sigma_y^t} \tag{12}$$

with $\sigma_y^c$ and $\sigma_y^t$ respectively the yield stress in compression and in tension.

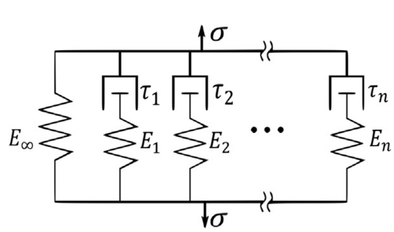


**Fig. 1.** Schema of generalised Maxwell's model.