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# A general sandwich-type model for adhesive joints with composite adherends

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ABSTRACT

Efficient analyses of adhesive joints are very important for pre-dimensioning or optimization processes. In this work a new model for adhesive joints is presented that can be applied to various joint configurations, as e.g. single lap joints, T-joints, L-joints or reinforcement patches. This is done by means of a two-dimensional, plane-strain model of the overlap region of the adhesive joint that is loaded by arbitrary section forces and moments — a so called general sandwich-type model. To account for shear deformations of the adherends, First Order Shear Deformation Theory is used. Arbitrarily laminated adherends are considered including asymmetric laminates with bending-extension coupling. The model yields a shear and peel stress distribution in the thin adhesive layer that is constant over the thickness. The results are compared to Finite Element Analyses for several joint configurations and show good agreement. A closed-form analytical solution is given for the case of identical adherends without bending-extension coupling. The general solution can be obtained with very low numerical effort. To allow for a ready use of the present solution an efficient MATLAB implementation is provided as supplementary material to this paper.



## 1. Introduction

With the ongoing development of adhesives and the widespread use of fibre reinforced plastics (FRP) adhesive bonds are increasingly used. Correspondingly, a solid understanding of the mechanical behaviour is required to design and size adhesive bonds. In this regard the knowledge about the load transfer in the joint and the resulting stress distribution in the adhesive layer and the adherends is an essential point.

So far, many publications have been devoted to the analysis of stresses in adhesive bonds. Of course detailed Finite Element (FE) analyses allow for an accurate analysis of the stress distribution in arbitrarily shaped adhesive bonds [1], but they require proper modelling and have substantial computational costs. To allow for an efficient study of the stress distribution and its parameter dependencies more efficient approaches are needed. Especially in pre-dimensioning stages or optimization processes such approaches are of high importance. The earliest closed-form analytical models of adhesive bonds are those of Volkersen [2] and of Goland and Reissner [3] which apply to single lap joints (SLJ). Both models use a weak interface formulation that describes the adherends as beams and the adhesive layer as smeared springs. In the model of Volkersen only shear stresses are considered, whereas the model of Goland and Reissner yields a shear and peel stress distribution. Following the same concept many different models have been proposed in literature. A refined analysis of the effect of the adherend bending was proposed by Hart-Smith [4], Renton and Vinson [5] considered composite adherends, Ojalvo and Eidinoff [6] studied the effect of the adhesive thickness and an extension of the Goland-Reissner model was proposed by Bigwood and Crocombe [7]. In this model only the overlap region of the bond is considered, which allows for the analysis of various joint configurations, as e.g. the single lap joint, L-joints and T-joints. In the following such models will be referred to as general sandwich-type analyses as they model the joint only as a three layer structure. To account for shear deformations as they become important in fibre reinforced plastics (FRP) adherends, the First Order Shear Deformation Theory (FSDT) has been employed in joint analyses, e.g. by Yang and Pang [8] for single lap joints, by das Neves et al. [9] for single lap joints with mixed adhesives or by Tsai, Morton and Oplinger [10] for single and double lap joints. Whereas the analysis by Yang and Pang also includes the effect of bending-extension coupling in asymmetrically laminated adherends. In this analysis a system of differential equations of 12th order must be solved by use of Fourier series. In the past decades many more refined models have been published [11–18]. A comprehensive overview of available closed-form analytical models is given by da Silva et al. [19]. However, for the case of laminated adherends up to now no general sandwich-type model that can be used to analyse various joint configurations has been reported. The aim of the present work is to

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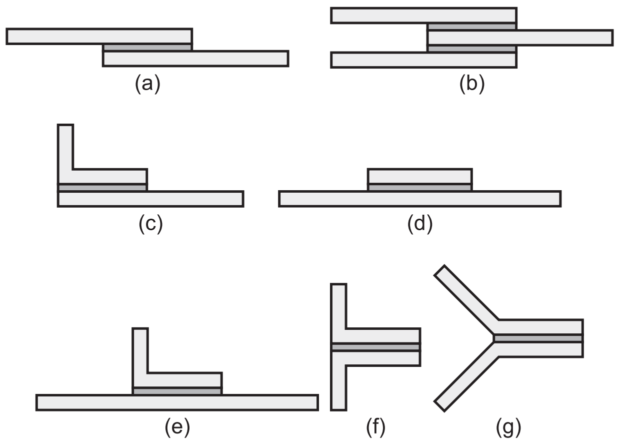


**Fig. 1.** Examples of adhesive joint configurations that are covered by the present model. Arbitrary in-plane loading can be considered. ((a) single lap joint, (b) double lap joint, (c) L-joint, (d) reinforcement patch, (e) T-joint, (f) peel joint, (g) inclined peel joint)

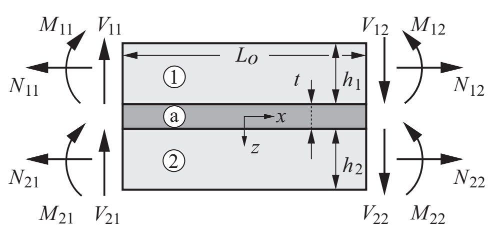


**Fig. 2.** Considered sandwich-type element with general loading.

provide such an efficient generally applicable model for the stress analysis of bonded joints with laminated adherends.

The derivation of the governing equations and the boundary conditions are given in detail in section 2. In section 3 the results of the present model are discussed and compared to reference solutions obtained with detailed FE analyses of several structural situations.

## 2. Governing equations

A two-dimensional, plane-strain general sandwich-type model of the overlap region of an adhesive joint with arbitrary section forces and moments is used to derive an analysis applicable to various commonly used joint configurations, e.g. single lap joints, T-joints, L-joints or reinforcement patches, see Fig. 1. The concept of general sandwich-type models allows for analyses of arbitrary joint configurations that have an overlap region like the considered sandwich-type element shown in Fig. 2. As shown in the results section double lap joints can also be modelled by observing the respective symmetry conditions.

The layered sandwich-type model representing the overlap region is introduced in Fig. 2. To allow for analysis of various structural situations all possible section forces are considered at each end of the top and bottom layer: a bending moment $M$, a shear force $V$ and a normal force $N$. The thicknesses of the top and the bottom layers, that represent the adherends of the adhesive bond, are denoted by $h_1$, $h_2$, respectively. The thickness of the middle layer, representing the adhesive layer is denoted by $t$. The width of the whole sandwich-type element is $b$ and the length of the overlap region is $L_O$. In the following a Cartesian coordinate system $x$, $z$ with its origin in the centre of the adhesive layer will be used. The corresponding displacements are denoted $u$, $w$. The index 1 is used for the upper adherend, the index 2 for the lower adherend

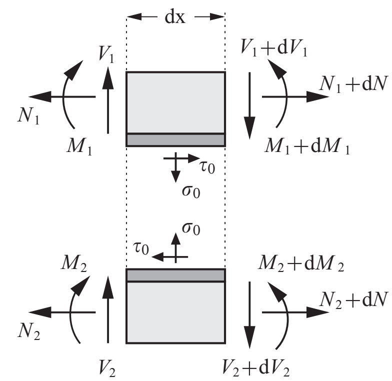


**Fig. 3.** Free body cut of an element with length d$x$ of the sandwich.

and the index $a$ is used for the adhesive layer. We will use $(\cdot)'$ to denote a derivative with respect to $x$ in the following calculus.

Let us start from the free body cut of an element with length d$x$ of the sandwich as depicted in Fig. 3. In the adhesive layer the shear stress $\tau_0(x)$ and the peel stress $\sigma_0(x)$ are considered. The conditions for equilibrium of force and moment for the upper adherend read

$$N_1' + \tau_0 = 0, \tag{1}$$

$$V_1' + \sigma_0 = 0, \tag{2}$$

$$M_1' - V_1 + \frac{(h_1 + t)\tau_0}{2} = 0 \tag{3}$$

and analogously for the lower adherend:

$$N_2' - \tau_0 = 0, \tag{4}$$

$$V_2' - \sigma_0 = 0, \tag{5}$$

$$M_2' - V_2 + \frac{(h_2 + t)\tau_0}{2} = 0. \tag{6}$$

It is to note that in these relations the thickness of the adhesive layer is considered, whereas it is neglected in most of the common analyses of adhesive joints.

The consideration of the shear deformation of the adherends is important when the adherends have relatively low transverse and normal stiffnesses. This is especially the case for FRP adherends. Then a theory that uses the Kirchhoff-Love assumptions, as e.g. the Classical Laminated Plate Theory (CLPT), is not sufficient to model the deformations of the adherends. Therefore, in this work the First Order Shear Deformation Theory (FSDT) (see e.g. [20]) is used to model the adherends. Using $\hat{z}$ as a coordinate in the center of the adherends the kinematics in the adherends are:

$$u(x,z) = u_0(x) + \hat{z}\psi(x), \tag{7}$$

$$v = 0, \tag{8}$$

$$w(x) = w_0(x), \tag{9}$$

$$\varepsilon_x = u_0'(x) + \hat{z}\psi'(x), \tag{10}$$

$$\gamma_{xz} = w_0'(x) + \psi(x) \tag{11}$$

where $u_0, w_0$ denote the displacements of a point on the plane $\hat{z} = 0$ and $\psi$ the rotation of a transverse normal about the y-axes. As in the whole analysis the condition of plane-strain is presumed, the section forces and moments are related to the mid-plane strain and curvature of the adherends in the following way