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Adhesive stresses in axially-loaded tubular bonded joints—Part II: Development of an explicit closed-form solution for the Lubkin and Reissner model

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ABSTRACT

The literature presents several analytical models and solutions for single- and double-lap bonded joints, whilst the joint between circular tubes is less common. For this geometry the pioneering model is that of Lubkin and Reissner, Transactions of The ASME 78 (1956) 1213–1221, in which the tubes are treated as cylindrical thin shells subjected to membrane and bending loading, whilst the adhesive transmits shear and peel stresses which are a function of the axial coordinate only. Such assumptions are consistent with those usually adopted for the flat joints. A former investigation has shown that the L–R model agrees with FE results for many geometries and gives far better results than other models appeared later in the literature. The aim of the present work is to obtain and present an explicit closed-form solution, not reported by Lubkin and Reissner, which is achieved by solving the governing equations by means of the Laplace transform. The correctness of the findings, assessed by the comparison with the tabular results of Lubkin and Reissner, and the features of this solution are commented.

1. Introduction

The literature survey carried out in the first part of this study [1] and the related comparison with finite element (FE) results have evidenced that, among the known models of the tubular bonded joints under axial loading [2–9], only the one by Lubkin and Reissner [2] gives a truthful distribution of the peel stress in the overlap, while the shear component is predicted correctly in all models. Moreover, the FE results evidence that the peel and shear stresses are the most important components; the remaining ones, namely the axial and hoop stresses, have similar magnitude and are about one half of the peel stress.

On the basis of these findings, the aim of this work is reconsidering the model by Lubkin and Reissner to make up for its practical shortcoming, which is the lack of an explicit closed-form solution. The set of differential equations is solved by means of the Laplace transform, with a procedure modified to cope with the issue of dealing with a boundary problem (the known conditions are applied at the ends of the overlap) instead of an initial value problem (as in typical dynamic problems). The result is an explicit formula for the solution, which evidences the differences with respect to the flat lap joint and allows for direct calculation of the stresses.

2. Lubkin and Reissner model

The model by Lubkin and Reissner [2], for which a brief description has already been given in the first part [1] of the present study, is reviewed here in more detail. Fig. 1 shows the shape of the joint as well as the geometrical and material properties. Considering first the tubes (subscripts 1, 2), a_1 and a_2 are the mean radii of the walls, E_1 and E_2 are the Young moduli, ν_1 and ν_2 are the Poisson's ratios. Regarding the adhesive (subscript a, when adopted), a is the mean radius of the layer, η is the thickness, E_a is the Young modulus, G_a is the shear modulus. The axial force loading the joint is F, the overlap length is $2c$; thus, having set the origin at midspan, the axial coordinate x varies in the range $\pm c$. Also the normalized coordinate z is adopted, varying between 0 (left end) and 1 (right end). With reference to Fig. 2, accounting for axial (T_1, T_2), transverse (V_1, V_2) and hoop (N_1, N_2) forces per unit length, bending moments (M_1, M_2) per unit length,

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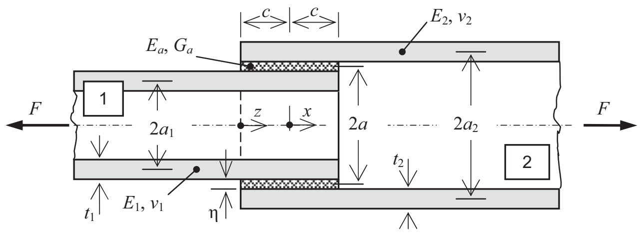

Fig. 1. Schematic of the tubular joint (also the related elastic constants are shown).

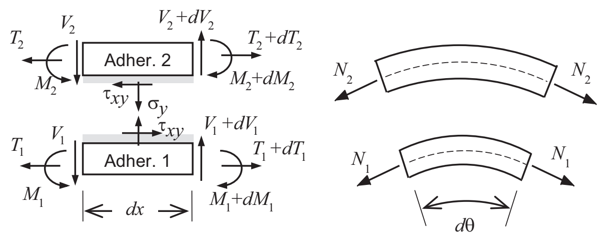

Fig. 2. Elementary free body diagrams for the joint.

peel (σ_y) and shear (τ_{xy}) stresses in the adhesive, the following equilibrium equations can be written for the two adherends:

$$a_1\frac{dT_1}{dx}+a\tau_{xy}=0;\; a_2\frac{dT_2}{dx}-a\tau_{xy}=0 \tag{1a, b}$$

$$a_1\frac{dV_1}{dx}+a\sigma_y-N_1=0;\; a_2\frac{dV_2}{dx}-a\sigma_y-N_2=0 \tag{2a, b}$$

$$a_1\frac{dM_1}{dx}-a_1V_1+a\frac{t_1}{2}\tau_{xy}=0;\; a_2\frac{dM_2}{dx}-a_2V_2+a\frac{t_2}{2}\tau_{xy}=0 \tag{3a, b}$$

The following equations of axial, hoop and bending deformability can be respectively written, which involve the longitudinal (u_1, u_2) and transverse (v_1, v_2) displacements at the mean radii of the tubes:

$$\frac{du_1}{dx}=\frac{T_1-\nu_1N_1}{E_1t_1};\; \frac{du_2}{dx}=\frac{T_2-\nu_2N_2}{E_2t_2} \tag{4a, b}$$

$$\frac{v_1}{a_1}=\frac{N_1-\nu_1T_1}{E_1t_1};\; \frac{v_2}{a_2}=\frac{N_2-\nu_2T_2}{E_2t_2} \tag{5a, b}$$

$$\frac{d^2v_1}{dx^2}=-\frac{M_1}{D_1};\; \frac{d^2v_2}{dx^2}=-\frac{M_2}{D_2} \tag{6a, b}$$

where D_1, D_2 are the bending stiffnesses, defined as $D_i=E_it_i^3/12(1-\nu_i^2)$, with $i=1, 2$.

The peel and shear stresses in the adhesive are related to displacements of the outer surface of tube 1 and inner surface of tube 2:

$$\sigma_y=\frac{E_a}{\eta}(v_2-v_1) \tag{7}$$

$$\tau_{xy}=\frac{G_a}{\eta}(u_{2,in}-u_{1,ou}) \tag{8}$$

It must be noted that in Eq. (7) the surface displacements coincide with those of the mean surfaces, whilst in Eq. (8), accounting for the membrane and bending behaviour, the displacements are $u_{1,ou}=u_1-(dv_1/dx)(t_1/2)$ for the outer surface of tube 1 and $u_{2,in}=u_2+(dv_2/dx)(t_2/2)$ for the inner surface of tube 2.

Thus, the problem involves in total fourteen equations – from (1a,b) to (6a,b), plus (7) and (8) – in the fourteen unknowns T_1, V_1, N_1, M_1, u_1, v_1; T_2, V_2, N_2, M_2, u_2, v_2; σ_y, τ_{xy}, which are all a function of x. In the solution procedure depicted in [2], by means of a sequence of manipulation V_i, N_i, M_i, u_i ($i=1,2$) and σ_y are eliminated; moreover, noticing that for the global axial equilibrium the condition $2\pi(a_1T_1+a_2T_2)=F$ must hold, an auxiliary unknown T_0 is assumed such that

$$aT_0=a_2T_2-a_1T_1 \tag{9}$$

and, from Eq. (1a,b),

$$\frac{dT_0}{dx}=2\tau_{xy} \tag{10}$$

Therefore, the axial forces per unit length T_1, T_2 and the shear stress in the adhesive τ_{xy} can be expressed as a function of such auxiliary unknown T_0; by mathematical manipulations a set of three simultaneous differential equations is obtained in the three unknowns v_1, v_2