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Procedure for studying the repeated contacts and separations in an axial impact involving a non-uniform elastic bar

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A B S T R A C T

The generation of different waveforms by axial impacts involving a non-uniform bar can be applied in percussive drilling to increase wave energy transfer. It is a good method for obtaining the force and velocity histories at the impacted ends by introducing a delta function, based on 1-D theory. However, they could not be directly predicted, considering that the two impacted ends may be separated from one another, and a clearance is therefore generated between them. In addition, the changing-status of the non-linearities given the repeated separations and contacts is discussed in this paper. The conditions of separation and contact are provided to present a procedure for predicting the parameters at the impacted ends throughout the non-linear process, and the boundary conditions are also considered. As illustrated in an example impact model, the self-recontact behavior resulting from the wave motion occurs, and the possibility for repeated contacts and separations phenomenon is revealed.

1. Introduction

The efficiencies of technological processes such as percussive drilling depend greatly on the history of the generated impact forces. Incident waves derived from the impact force interact with the rocks' penetration resistances. Thus, the energy transfer efficiency has been studied, for example as in Refs. [1–3], considering the generation of reflected waves. Optimal incident waves with exponentially increasing amplitudes have been obtained using the Schwarz inequality because the loading and unloading penetration resistances were taken into account [4], which were not considered in earlier work [5]. Generally, making measurements of sufficient quality at the impacted end-faces may not be feasible or accurate. However, sometimes parameters can be evaluated using data measured at some distance from the impacted end [6,7]. Therefore, it is of significance to obtain such histories by prediction, in which no test data are needed.

In terms of percussive (top hammer) drilling, a hammer is loaded at a certain speed to apply an axial impact between a long rod at rest, and the initial kinetic energy in the hammer is partially or completely transferred into the rod. We are potentially concerned with the maximum or minimum energy transfer. For two uniform elastic bars, it is easy to show that the status of the separation or contact depends on the body-to-body characteristic impedances ratios. For an axial impact involving a non-uniform bar (at least one of the two bars is non-uniform), separations cause the energy transfer process to be suspended, and the residual potential and kinetic energies are isolated inside the hammer. Moreover, the potential for repeated separations and contacts should be investigated. Cases of elastic [8] and viscoelastic [9] impacts in which contact/separation occurs just once have been presented in studies. The repeated impacting behavior arising from a cantilever beam driven by a periodic force against a rod-like stop has been investigated [10]. The "successive collisions" phenomenon generated by the axial impact between a rod and a concentrated mass has been theoretically proven by Gao [11] and Escalona et al. [12]; furthermore, the dynamic substructure method has been shown to be sufficiently accurate [13]. The propagation laws of longitudinal waves in a non-uniform bar are needed to understand the mechanisms of separation and later contact. Crucial theorems and properties regarding the propagation of the incident, reflected and transmitted waves as well as energy transfer have been developed from several proofs by Andersson and Lundberg [14]. For a given incident wave, there is an optimum variation in the characteristic impedance of a non-uniform bar that maximizes the energy transfer [15,16] and vice versa [17]. Different transmission-equivalent junctions that contribute the same transmitted waveform for the same arbitrary incident wave have been discussed [18]. The impulse response method has been verified to be effective in determining the normal force and velocity at the impacted end [8,9], and synthesizing the characteristic impedance of a hammer in an attempt to generate the anticipated history of an impact force [19].

In this paper, a general impact model is selected for analysis. After the separation for the first time, the re-contact behavior plays an important role in accounting for the later repeated contacts and

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Nomenclature

Parameters and variables

E	Young's modulus
$Z_i(\xi)$	characteristic impedance of body "i"
A_i	cross-sectional area of body "i"
c	wave speed
t	time
d_i	diameter of body "i"
x	axial coordinate
ρ	density
ν	Poisson ratio
ξ	$\xi = \int_0^x dy/c(y)$
$\delta(t)$	delta function
V_{ini}	initial velocity of body 1
l	finite length of bar
u	gap between two IEs
θ, τ	dimensionless parameters governing the shape of the hammer generated exponential load
h	transit time needed for a wave to pass through every segment of the piece-wise bar
G_{iI}^f, G_{iI}^v	velocity response (positive in the direction from IE to NIE) at IE by excitations of force $\delta(t)$ at IE and force, velocity $0(t)$ at NIE of body "i" without V_{ini}
G_{iNI}^f, G_{iNI}^v	velocity response (positive in the direction from IE to NIE) at IE by excitations of force $0(t)$ at IE and force, velocity $\delta(t)$ at NIE of body "i" without V_{ini}
f_I	actual impact force (positive in compression)
f_{iNI}, v_{iNI}	boundary condition constrained in form of force (positive in compression), velocity (positive in the direction from IE to NIE) at NIE of body "i" without V_{ini}
v_{iI}	actual velocity (positive along V_{ini} direction) at the IE of body "i"
$\xi_{\max}$	maximum upper bound of ξ of hammer generated exponential load
ξ^a	actual upper bound of ξ
$\boldsymbol{H}$	transfer matrix
$B, \bar{B}, D$	elements of $\boldsymbol{H}$

Abbreviations

IE	impacted end
NIE	non-impacted end

separations that can occur often. A procedure based on the impulse response is developed to stepwise determine the normal impact forces and velocities at the impacted ends throughout the non-linear process, based on the (beginning) separation and contact conditions. The boundary conditions as an influencing factor are considered to solve more general impact problems. It is helpful to determine the beginning times of the separations and contacts, or even the body-to-body energy conversions through the multiple impacts to learn more about the complete rebound phenomenon, which is of importance for percussive drilling or rock and other brittle (or semi-brittle) materials.

2. Methods for solving the non-linear problem

2.1. Theoretical fundamentals

For elastic wave propagation along the axial coordinate $x \ge 0$ in a straight non-uniform bar with Young's modulus E and density ρ, the wave propagation speed is $c = (E/\rho)^{1/2}$. By introducing the wave propagation time $\xi = \int_0^x dy/c(y)$, the cross-sectional area $A(\xi)$ and characteristic impedance $Z(\xi) = AE/c$, which represent the variables of the cross-section that is reached by a disturbance from IE in time ξ, are functions of ξ.

Primarily, a generalized function $\delta(t)$ can be defined to satisfy $\int_{-\infty}^{+\infty} \delta(t)dt = 1$ and $\delta(t) = 0\,(t \ne 0)$.

Impact model is shown in Fig. 1; generally, body 1 is the part that moves with an initial velocity V_{ini}, body 2 is the stationary target part prior to impact. Each body's coordinate ξ increases along the direction of the arrow, and the origin is at the impacted end (IE). The actual impact force $f_I(t)$ is considered to be the common impact force generated at the IEs considering the possibility of separation and is positive in compression. For the two bodies with arbitrary variations in characteristic impedance, the non-uniformity problems can be solved using the discretization method presented in Ref. [14], such that the non-uniform bodies consist of uniform segments with constant characteristic impedances instead.

Each non-impacted end (NIE) could be specified by boundary condition in the form of $f_{iNI}(t)$ or $v_{iNI}(t)$ that does not include V_{ini} in body "i". In the context of 1-D elastic theory, the velocity added to V_{ini} at the IE under the influence of $f_{iNI}(t)$ or $v_{iNI}(t)$ is treated as independent of $f_I(t)$ and conforms to a full superposition in conjunction with $f_I(t)$, viz., the two pairs of convolution relations can be expressed as

$$v_{1I} = V_{ini} - G_{1I}^f * f_I - G_{1NI}^f * f_{1NI}, v_{1I} = V_{ini} - G_{1I}^v * f_I - G_{1NI}^v * v_{1NI}, \tag{1.a}$$

and

$$v_{2I} = G_{2I}^f * f_I + G_{2NI}^f * f_{2NI}, \qquad v_{2I} = G_{2I}^v * f_I + G_{2NI}^v * v_{2NI} \tag{1.b}$$

where v_{iI} is the actual velocity response at the IE of body "i" and is positive along the V_{ini} direction. G_{iNI}^f and G_{iI}^f, which are the velocity responses at the IE, are excited by impulse force $\delta(t)$ at the NIE and IE alone, respectively. Similarly, G_{iI}^v and G_{iNI}^v are excited by the impulse force $\delta(t)$ at the IE and impulse velocity $\delta(t)$ at the NIE