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A method to represent impacted structures using scaled models made of different materials

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ABSTRACT

A technique that deals with scaling of structures under dynamic loads is studied here. It is assumed a model whose material is different from the prototype so generating incomplete similarity. In order to handle this problem, a method was developed that allows the use of different densities and mechanical properties for replica and full-size structures. As a means to verify the technique, an analytical solution for beams under impulsive load and numerical simulations of scaled plates subjected to dynamic loads is explored. The theoretical solution shows that the structure density can play an important role. The complete similarity can only be achieved if the density scaling factor is taken into account, mainly for structures in which the inertia is comparable. The numerical investigations show that discrepancies between the scaled variables of model and prototype are substantially reduced when the present method is applied, even for material as different as steel and magnesium. The limitations caused by the use of different materials for model and prototype are thoroughly discussed.

1. Introduction

The use of models to represent the actual response of a prototype is an important tool in many engineering applications. It is well established in many investigations [1–3] that the direct application of standard similarity laws to impact events impairs one of obtaining accurate information of the prototype (real size structure) from the model (structure scaled by a factor) response. This behaviour is usually attributed to the non-regular scalability of phenomena like gravity, material failure and strain-rate hardening.

Oshiro and Alves [4] presented a way to deal with the influence of the strain-rate hardening on the non-scalability of structural impact models by modifying the scaling factor for the initial striker velocity. This allowed taking strain-rate effect into account for a rigid-perfectly-plastic material with its viscoplatic behaviour defined by a power law. Hence, the same deformed geometry pattern for both prototype and model could be obtained. Mazzariol et al. [5] enhanced the formulation presented by Oshiro and Alves and imposed arbitrarily striker velocities, compensating the input energy also by modifying the impact mass.

Alves and Oshiro [6] proposed a method that is able to deal with mild steel prototype using less strain-rate sensitive models; the initial conditions are calculated according to expected structure response. Cho et al. [7] proposed an approach that requires experimenting model and prototype made of the same material to obtain scaling correction factors. This can be troublesome in experimental applications since the making of structures as well as their material properties are often scale-dependent. The idea of using models of slightly different material was briefly investigated with experiments by Oshiro et al. [8], incorporating to the scaling laws a factor that relates differences in material flow-stress between model and prototype. The authors assumed same viscoplastic properties and density for model and prototype.

Limitations of experimental nature motivated Westine and Mullin [9] to use models made of different materials to capture the behaviour of a prototype under hypervelocity impact. Their study, however, deals with fragmentation and cannot be directly applied to structures loaded in a way that no material failure occurs.

Recently, researchers presented studies on scaling laws relating models to the behaviour of prototype under impact loading for sandwich structures [10], different isotropic materials [11], dynamic failure [12] and impulsive loading [13]. In spite of all these studies, it has been an open problem to obtain information of the prototype behaviour from scaled models, especially when, due to manufacturing, costs or experimental restrictions, they are made of different materials.

In the present investigation, scaling laws relating prototype and scaled models with differences in materials properties, specifically the density, yield stress, strain hardening and strain-rate hardening will be derived for structures under dynamic loads. These laws will

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be shown to give exact scaling for rigid-perfectly-plastic beams under impulsive loading when using appropriate viscoplastic equation. They are also applied numerically to a circular plate loaded by a pulse load and by the impact of a mass. The given examples consider a prototype structure made of an expensive low density material (magnesium alloy AZ31B), while the models are made of materials like mild steel or aluminium alloy.

In what follows, Section 2 describes the background of the here developed distorted similarity approach, while Section 3 shows the comparison between existent and developed approaches via numerical results. Section 4 discusses the results, with Section 5 closing the article.

2. Distorted similarity and structural impact

The technique in which a model (scaled by a factor β) is used to infer the prototype behaviour is termed *similarity*, *scaling* or *similitude*. This method has been extensively studied and applied in many works [14–22]. For impact phenomena, the main variables and their scaling factors are long known and are listed in Table 1. In order to achieve perfect similarity, the Π theorem [23] asserts that all model predominant dimensionless numbers, Π_i, must be equal to the corresponding prototype dimensionless numbers:

$$(\Pi_i)_m = (\Pi_i)_p, \tag{1}$$

with m and p referring to model and prototype, respectively.

As mentioned, structures under severe dynamic loads usually do not follow standard scaling laws due to effects such as material strain-rate sensitivity, material failure, thermal response and gravity. When a single geometric scaling factor is not capable of relating a prototype to a model, it is necessary to define other scaling factors, an approach called *distorted similarity*.

2.1. Previous approaches

In order to compensate for strain-rate effects, Oshiro and Alves [24] defined the velocity factor:

$$\beta_V = \sqrt{\frac{\sigma_{d_m}}{\sigma_{d_p}}} = \sqrt{\frac{f(\dot{\varepsilon}_m)}{f(\dot{\varepsilon}_p)}}, \tag{2}$$

with σ_d and $\dot{\varepsilon}$ being the dynamic stress and strain-rate, respectively. Equation (2) does not specify the viscoplastic law to be used, $f = f(\dot{\varepsilon})$, and Oshiro and Alves [24] used the Cowper–Symonds equation:

$$\frac{\sigma_d}{\sigma_0} = \left[1 + \left(\frac{\dot{\varepsilon}}{D}\right)^{1/p}\right], \tag{3}$$

being σ_0 the flow stress, p and D, viscoplastic material properties. From Eqs. (2) and (3), it follows for model and prototype made of the same material that [24]

$$\beta_V = \sqrt{\frac{\left[1 + (\dot{\varepsilon}_m/D)^{1/p}\right]}{\left[1 + (\beta\dot{\varepsilon}_m/\beta_V D)^{1/p}\right]}}, \tag{4}$$

which has the shortcome of being necessary to know the model strain-rate, $\dot{\varepsilon}_m$. This was overcame in Oshiro and Alves by adopting the Norton equation to describe the material viscoplastic behaviour:

$$\sigma_d = \sigma_0\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ref}}\right)^q, \tag{5}$$

yielding the velocity scaling factor

$$\beta_V = \beta^{q/(q-2)}, \tag{6}$$

with q being a material constant.

The chiefly advantage of Eq. (6) is that the impact velocity factor does not require one to know, *a priori*, the model or prototype behaviour. It is only necessary to know the material parameter q to evaluate the impact velocity one should apply to the model such that the prototype behaviour can be inferred. A variant of Eq. (6) was proposed in Ref. [25]. In this case, it was assumed $\beta_V = 1$ and the impact mass factor was calculated using $\beta_G = \beta^{3-q}$. It is also possible to relax both impact mass and velocity [5], what leads to important experimental advantages.

The problem of different materials for model and prototype was studied by Alves and Oshiro [6] who proposed an approach based on a variation of Eq. (2). In this case, the model was made of aluminium (low sensitivity to strain-rate) and the prototype of steel (high strain-rate sensitivity), with the velocity factor given by:

$$\beta_V = \sqrt{\frac{\sigma_{0_m}}{\sigma_{0_p}\left[1 + \left(\frac{\beta}{\beta_V}\frac{\dot{\varepsilon}_m}{D}\right)^{1/p}\right]}}, \tag{7}$$

when adopting the Cowper–Symonds constitutive law and with the model response, $\dot{\varepsilon}_m$, being obtained numerically or theoretically.

Mazzariol and Alves [11] revisited Ref. 24 by considering that model and prototype are made of material with different yield stresses, $\beta_{\sigma_0} \neq 1$, such that the impact mass factor now reads:

$$\beta_G = \beta^3\beta_{\sigma_0}, \tag{8}$$

being β_{σ_0} the flow stress scaling factor. This equation leads to a distortion of β_V, which is reduced when the model is stronger than the prototype. In all those approaches, the density of model and prototype is not considered in the correction. It seems worthwhile to expand these scaling methodologies by considering material with different densities, as now examined.

2.2. Different materials

Using the VSG basis (initial Velocity, V_0, dynamic Stress, σ_d, and impact mass, G), the following dimensionless numbers are obtained [4]:

$$\Pi_1 = \left[\frac{a^3 G}{V_0^4 \sigma_d}\right], \Pi_2 = \left[\frac{t^3 \sigma_d V_0}{G}\right], \Pi_3 = \left[\frac{\delta^3 \sigma_d}{G V_0^2}\right], \\ \Pi_4 = \left[\frac{\dot{\varepsilon} G^{1/3}}{(\sigma_d V_0)^{1/3}}\right] \text{and } \Pi_5 = \left[\frac{\sigma}{\sigma_d}\right]. \tag{9}$$

Since a dimensionless number must be the same for model and prototype, it follows that $(\Pi_3)_m = (\Pi_3)_p$, or

Table 1
Factors relating the model variables to the prototype in the MLT (Mass–Length–Time) basis.

Variable	Factor	Variable	Factor
Length, L	β	Time, t	$\beta_t = \beta$
Displacement, δ	$\beta_\delta = \beta$	Velocity, V	$\beta_V = 1$
Impact mass, G	$\beta_G = \beta^3$	Strain-rate, $\dot{\varepsilon}$	$\beta_{\dot{\varepsilon}} = 1/\beta$
Strain, ε	$\beta_\varepsilon = 1$	Acceleration, a	$\beta_a = 1/\beta$
Stress, σ	$\beta_\sigma = 1$	Energy, E'	$\beta_{E'} = \beta^3$
Force, F	$\beta_F = \beta^2$	Density, ρ	$\beta_\rho = 1$