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Dynamic fracture simulations using the scaled boundary finite element method on hybrid polygon–quadtree meshes

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ABSTRACT

In this paper, we present an efficient computational procedure to model dynamic fracture within the framework of the scaled boundary finite element method (SBFEM). A quadtree data structure is used to discretise the domain, and 2:1 ratio between the cells is maintained. This limits the number of patterns in the quadtree decomposition and allows for efficient computation of the system matrices. The regions close to the boundary are discretised with arbitrary sided polygons so as to facilitate accurate modelling of the curved boundaries. The stiffness and the mass matrix over all the cells are computed by the SBFEM. Moreover, the semi-analytical nature of the SBFEM enables accurate modelling of the asymptotic stress fields in the vicinity of the crack tip. An efficient remeshing algorithm that combines the quadtree decomposition with simple Boolean operations is proposed to model the crack propagation. The remeshing is restricted only to a small region in the vicinity of the crack tip. The efficiency and the convergence properties of the proposed framework are demonstrated with a few benchmark problems.

1. Introduction

Computational modelling of crack propagation is a challenging task that usually necessitates the availability of: (a) an efficient remeshing algorithm, (b) an accurate numerical method and (c) significant amount of computational resources. The finite element method (FEM) has achieved reasonable success in this area. Early crack propagation simulations with the FEM reported by Wawrzynek et al. [1–3] employed the computational software FRANC2D developed by the Cornell group. A three-dimensional version of the software FRANC3D has also been developed and applied to fracture problems in three dimensions [4,5]. Over the years, considerable effort has been invested to develop methods to improve the computational modelling aspects of crack propagation within the framework of the FEM, for example: (a) the development of special elements [6–8]; (b) high quality mesh generators [9,10] that enable the FEM to adapt to changing mesh topology during crack propagation; (c) adaptive refinement techniques [11,12] that preserve the quality of the finite elements generated after remeshing with an aim to improve the accuracy of the FEM solutions; and (d) smeared crack approach [13] where the crack is modelled as a limiting case of two singular lines which tend to coincide with each other. Recently, Areias and Rabczuk [14–17] combined the smeared cracking approach with local remeshing for finite strain problems. This combination alleviates the need to determine the correct characteristic length as required in the original smeared crack approach. To reduce the computational time, adaptive refinements are usually preferred over a uniform mesh refinement. Compared with the conforming refinements, quadtree/octree meshes [18] are particularly easy to implement. A typical quadtree/octree meshes have few elements with additional nodes, called 'hanging nodes'. There are problems associated with the incompatible displacement field introduced by the hanging nodes. While remedial techniques have been developed, e.g. references 19 and 20, the use of quadtree/octree meshes within the framework of the FEM is still not widespread, especially for crack propagation problems or problems with complex boundaries.

Another school of thought in the computational modelling of crack propagation advocates the implicit representation of the crack paths. The meshless methods [21–23] and the extended/generalised FEM (XFEM/GFEM) [24,25] both fall under this school of thought. In this approach, the crack surfaces are not explicitly discretised, and hence, frequent remeshing is not required. In the meshless methods, a set of paired nodes is used to model the crack surfaces. As the crack propagates, a new set of nodes is added to represent the new crack surfaces. Rabczuk and Belytschko [26] presented a simplified meshfree approach to treat evolving cracks. Within the framework, the crack can be arbitrarily oriented, and the

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growth of the crack is discretely activating the crack surfaces of individual particles. Later, Rabczuk and Belytschko [27] and Rabczuk et al. [28] extended the cracking particle approach to model evolving cracks in three dimensions and to model shear bands with cohesive surfaces, respectively. Many fracture problems have been successfully modelled by both the meshless methods and the XFEM [29–31]. Fries et al. [32] presented two approaches to treat hanging nodes within the framework of the XFEM. The first approach relied on deriving conforming shape functions for all the degrees of freedom present at the hanging nodes. The second approach constrained the fields at the hanging nodes to be the average of the neighbouring corner nodes of the hanging node. The basic idea in the XFEM/GFEM is to augment the conventional finite element basis with *a priori* known functions that span the singular stress field. This facilitates the representation of cracks/crack growth independent of the underlying finite element mesh.

In this paper, we develop a framework that centres about a hybrid polygon–quadtree based scaled boundary finite element method (SBFEM) to model dynamic crack propagation in isotropic materials. The SBFEM developed by Song and Wolf [33] is a semi-analytical method, known for its application in problems involving unbounded domains [34–36] and fracture [37–39]. The SBFEM is also sufficiently flexible that it can be formulated on any star convex polygon [40,41]. This enables the SBFEM to be directly adapted for computations in quadtree meshes. The displacement incompatibility between adjacent cells introduced by the presence of hanging nodes is eliminated by modelling each cell as a polygon irrespective of the presence of hanging nodes. The crack tip is modelled by the SBFEM. This enables the asymptotic stress field in the vicinity of the crack tip to be modelled accurately. As a crack propagates within the domain, the new boundaries that are generated by the new crack surfaces destroy the quadtree data structure. To this end, we propose a simple remeshing algorithm that combines the quadtree decomposition with simple Boolean operations. The remeshing is restricted only to a small region in the vicinity of the crack tip.

This manuscript is organised as follows. Section 2 provides a brief account of the SBFEM formulation and its salient features when adapted for computations with quadtree meshes. In Section 3, we introduce the application of the SBFEM within the context of hybrid polygon–quadtree meshes, which leads to an accurate and an efficient approach to model dynamic crack propagation. The computation of dynamic stress intensity factors, the remeshing algorithm and the mesh mapping procedure are also discussed. Section 4 shows the application of the hybrid polygon–quadtree SBFEM in some numerical benchmarks. The major conclusions are summarised in Section 5.

2. Quadtree-based scaled boundary finite element method

2.1. Scaled boundary finite element formulation

This section summarises the SBFEM for elasto-dynamics. Only the main equations necessary for the implementation of the method are explained. The reader is referred to references 42 and 43 for a more detailed account of the SBFEM for elasto-dynamics. The SBFEM can be formulated on star convex polygons with arbitrary number of sides [40]. The crux of the method relies on defining a centre from which the whole domain is visible. When modelling discontinuous surface, for example, crack, the polygon enclosing the crack should not form a closed loop, and the scaling centre should be placed at the crack tip. Any point (x_n, y_n) in the domain is related to the scaling centre coordinates, x_c and the coordinates of the nodes on the boundary x_b by the following equation [33]:

$$\mathbf{x}_n = \mathbf{x}_c + \xi\mathbf{N}(\xi_o, \eta_o)\mathbf{x}_b \tag{1}$$

The geometry of the polygon is scaled with respect to the scaling centre. A radial coordinate, ξ, satisfying $0 \le \xi \le 1$ is defined at the scaling centre. $\xi = 0$ at the scaling centre and $\xi = 1$ at the polygon boundary. Fig. 1 shows the SBFEM coordinate system on a generic star convex polygon. Each edge on the polygon boundary is discretised using one dimensional line elements with local coordinate, η, typical in the FEM.

Within a sector covered by a line element on the polygon boundary, the displacement field is expressed as

$$\mathbf{u}(\xi, \eta) = \mathbf{N}(\eta)\mathbf{\Psi}_n^{(u)}\xi^{-\mathbf{S}_n}\mathbf{c} \tag{2}$$

where $\mathbf{c}$ is the vector of integration constants and $\mathbf{N}(\eta)$ is the shape function matrix

$$\mathbf{N}(\eta) = \begin{bmatrix} N_1(\eta) & 0 & N_2(\eta) & 0 & \dots & N_m(\eta) & 0 \\ 0 & N_1(\eta) & 0 & N_2(\eta) & \dots & 0 & N_m(\eta) \end{bmatrix} \tag{3}$$

and m is the number of nodes. The matrices $\mathbf{\Psi}_n^{(u)}$ and $\mathbf{S}_n$ represent the deformation modes that can be represented by the polygon. They are obtained from a block diagonal Schur decomposition of the following Hamiltonian matrix [42]

$$\mathbf{Z} = \begin{bmatrix} \mathbf{E}_0^{-1}\mathbf{E}_1^{\mathrm{T}} & -\mathbf{E}_0^{-1} \\ \mathbf{E}_1\mathbf{E}_0^{-1}\mathbf{E}_1^{\mathrm{T}} - \mathbf{E}_2 & -\mathbf{E}_1\mathbf{E}_0^{-1} \end{bmatrix} \tag{4}$$

In Eq. (4), $\mathbf{E}_0$, $\mathbf{E}_1$ and $\mathbf{E}_2$ are the coefficient matrices that depend on the geometry and material properties of a polygon [42]

$$\mathbf{E}_0(\eta) = \int_{-1}^{1} \mathbf{B}_1^T(\eta)\mathbf{D}\mathbf{B}_2(\eta)|J|\,d\eta$$

$$\mathbf{E}_1(\eta) = \int_{-1}^{1} \mathbf{B}_2^T(\eta)\mathbf{D}\mathbf{B}_1(\eta)|J|\,d\eta$$

$$\mathbf{E}_2(\eta) = \int_{-1}^{1} \mathbf{B}_2^T(\eta)\mathbf{D}\mathbf{B}_2(\eta)|J|\,d\eta \tag{5}$$

where $\mathbf{B}_1(\eta)$ and $\mathbf{B}_2(\eta)$ are the standard SBFEM strain-displacement matrices, $|J|$ is the determinant of the Jacobian matrix required for coordinate transformation and $\mathbf{D}$ is the material constitutive matrix.

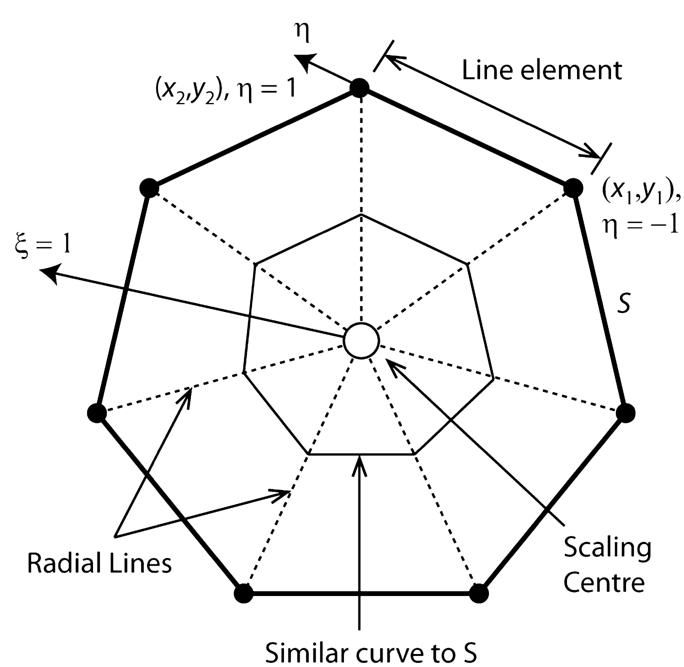

Fig. 1. Scaled boundary coordinate system on a generic star convex polygon.