International Journal of Impact Engineering 90 (2016) 165–176

Contents lists available at ScienceDirect

International Journal of Impact Engineering

journal homepage: www.elsevier.com/locate/ijimpeng

A robust algorithm for the generation of integration cells in Numerical Manifold Method

Yongchang Cai [a,b], Jie Wu [a,b,*]

[a] State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, Shanghai 200092, China
[b] Department of Geotechnical Engineering, Tongji University, Shanghai 200092, China

ARTICLE INFO

Article history:
Received 17 January 2015
Received in revised form 16 April 2015
Accepted 31 October 2015
Available online 10 November 2015

Keywords:
NMM
Finite cover system
Cracks
Integration cell
Tessellation approach

ABSTRACT

The finite cover system plays a critical role in Numerical Manifold Method (NMM) for the unified simulation of models from continuum to discontinuum. However, large amounts of computational geometries are usually involved in the traditional finite cover generation algorithm, which makes the process of finite cover generation a time consuming and error prone task and limits the wide applications of NMM. To achieve a simple and robust finite cover generation algorithm for NMM, a new method for the generation of integration cells including closed convex or concave polygons was developed in this paper as an important supplement to our recent work [Cai et al., 2013]. In the newly developed integration cells generation algorithm, with the help of pre-defined symbol functions and 3 different matrixes, nodes including vertexes and intersection points belonging to a same integration cell were first grouped together, and then listed in a anticlockwise manner to finally form the closed circuit. In this way, large amounts of computational geometries employed in the identification of integration cells were replaced with computational algebras. Several benchmarks were simulated to investigate the accuracy of the proposed integration cells generation algorithm and to demonstrate the robustness of NMM.

1. Introduction

The manners in which the additional degrees of freedom (DOFs) are inserted to simulate the discontinuities in displacement field are core properties for different numerical methods proposed to simulate cracks, like XFEM [2–5], meshless method [6–10], NMM [1,11,12], phantom node method [13–16], multiscale modeling method [17], and phase field method [18,19], etc. For NMM, which was proposed in 1991 by Dr. Shi [20,21], a dual covers system, including Mathematical Covers (MCs) and Physical Covers (PCs), is employed to insert additional DOFs for the simulation of models from continuum to discontinuum. Among them, the MCs are independent of the interior boundaries of the analysis domains and are kept unchanged during the whole computation process. The weight functions and the physical cover functions with DOFs are defined over MCs and PCs, respectively. While the interior boundaries exist in the analysis domain, the PCs would be cut to form sub-PCs where new physical cover functions with DOFs are added correspondingly to capture the displacement discontinuity. The dual cover system and the partition of PCs proposed in NMM make the insertion of the DOFs such a natural and concise manner. As a result, both simple cracks as well as complex cracks, especially like intersecting cracks and branched cracks, could be simulated in a unified framework. For more details of NMM, we refer to reference 22.

The finite cover system is critical for NMM to capture the discontinuity. However, the adopted finite cover system also gives rise to the generation of finite cover system for which a simple and stable computer algorithm is difficult to design and limits the wide application of NMM. Furthermore, similar as the condition in XFEM or phantom node method, cracks are allowed to arbitrarily pass through the mathematical elements (MMEs) in NMM, and the standard Gaussian quadrature rule, which is used for the numerical integration over continuous field function, may not adequately integrate the discontinuous field. To overcome this difficulty, several methods, such as tessellation approach [4,23,24], equivalent polynomials [25], and approach using moment fitting equations [26], have been proposed. A good review of these integration methods is given in Sudhakar and Wall [27]. Among these methods, tessellation approach and the moment fitting equations seem to be the most accurate and robust ways to evaluate the stiffness matrix especially for element arbitrarily cut by multiple cracks. For the implementation of these integration methods, the integration cells (closed convex or concave polygons) should be formed at first. Therefore, a simple and robust computer algorithm to implement the generation of finite cover system including the partition of PCs and the generation of integration cells is necessary for the practical application of NMM in engineering. To solve this problem, in the work

* Corresponding author. Department of Geotechnical Engineering, Tongji University, Shanghai 200092, China. Tel.: +86 021 65983809; Fax: +86 021 65985014.
E-mail address: jie.voo@gmail.com (J. Wu).

http://dx.doi.org/10.1016/j.ijimpeng.2015.10.015
0734-743X/

of Chen and Li [28], the mathematical mesh was regarded as the union of MMEs rather than MCs, and the finite cover system could be generated much more efficient.

For the generation (partition) of PCs, a simple and steady algorithm based on the symbol function was proposed in our recent work [1]. The computational geometries that are time consuming and error prone are largely avoided in this algorithm, and with the help of the value of symbol function, physical covers could be generated and connected to the corresponding ManiFold Elements (MFEs) in a robust manner.

Following our recent work [1], for the generation of integration cells, a robust algorithm is proposed based on the same symbol function in the paper. Large amounts of computational geometries involved in the process of 'identification of close bounded polygon' were replaced with computational algebras, which could be implemented obviously in a more stable manner.

With the generation algorithms for both of the PCs as well as the integration cells, the whole process of the generation of finite cover system of the NMM for the unified analysis of continuum and discontinuum problems could be consistently implemented in a simple and stable way. In addition, although the generation of the integration cells is proposed in the framework of NMM, this algorithm could also be used in the other numerical methods, like XFEM, phantom node method and Partition of Unity Method [29,30], which usually allow the cracks to pass through the interpolation elements. The proposed algorithm could offer a strong technique support for these methods and make its implementation more stable.

We start the investigation first with a brief introduction of NMM in section 2. This discussion will explain the basic manner in which the new physical covers and the new integration cells are generated, respectively. The natural and straightforward way to capture the discontinuous displacement field between cracks will also be highlighted in this section. The procedure of the new integration cells generation algorithm proposed in this paper will be introduced in detail in section 3. In order to show the consistency and to have a better realization of the whole generation process of finite cover system, the generation of the physical cover system proposed recently will be introduced after a slight revision in section 3 as well. In section 4, a comparison between NMM and FEM was first implemented based on a benchmark to investigate the accuracy and rate of convergence of the proposed numerical integration cells generation algorithm. Then several stationary linear elastic problems with simple and complex cracks are calculated to demonstrate the validity and the robustness of the proposed new integration cells generation algorithm.

2. Expression of discontinuous field in NMM

2.1. Mathematical cover system and weight function

Consider a rectangular analysis domain Ω with 3 internal discontinuity boundaries, represented by 2 lines l_i, l_j and a curve l_k as shown in Fig. 1. The intersection point of line l_i and l_j is ij.

As mentioned above, a dual cover system including MCs and PCs is employed by NMM for the unified simulation of continuum problem and discontinuum problems. Specially, for the analysis of discontinuous domain Ω, a mathematical cover system with 38 mathematical nodes was used (see Fig. 2). The mathematical cover system could either be uniform mesh or finite element mesh. For simplicity, the finite element mesh was used in this paper. As shown in Fig. 2, the MC for the node n_7 is the polygon with n_1, n_2, n_3, n_4, n_5, n_6 as its vertex, the MC for the node n_{14} is the polygon with n_8, n_9, n_{10}, n_{11}, n_{12}, n_{13} as its vertex, while the MC for the node n_{17} is the polygon with n_{18}, n_{10}, n_{11}, n_{15}, n_{16} as its vertex. In NMM, the MCs are independent of the internal boundaries and are kept unchanged during the whole simulation.

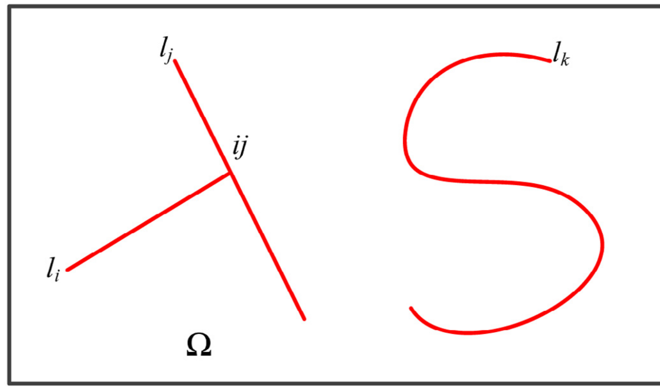

Fig. 1. Schematic representation of a solid body Ω with internal discontinuity boundaries l_i, l_j and l_k.

The weight function φ_i $(i = 1, 2, 3 \ldots m)$ (where m = 38 for this case) is defined over the mathematical cover MC_i $(i = 1, 2, 3 \ldots m)$. For example, the weight function for mathematical cover MC_{14} is φ_{14}, the weight function for mathematical cover MC_{17} is φ_{17}, and the weight function for MC_{17} is φ_7. The weight function φ_i $(i = 1, 2, 3, \ldots, m)$ satisfies the delta property and partition of unity. In NMM, the MMEs are the overlapping areas of MCs. For example, the MME $n_{12}n_{13}n_{14}$ is the overlapping part of the MCs including MC_{12}, MC_{13}, and MC_{14}, while the MME $n_7n_4n_5$ is the overlapping part of the MCs including MC_7, MC_4, and MC_5.

2.2. Physical cover system and physical cover function

Physical cover system is another cover system employed in NMM. The domain of PCs is completely the same as its corresponding MCs if it is not cut by cracks. For example, as shown in Fig. 3, the physical cover PC_{17} for node n_{17} is the same as its mathematical cover MC_{17} which is the polygon with nodes $n_{16}n_{15}n_{11}n_{10}n_{18}n_{17}$ as its vertexes. While a PC is completely cut by cracks, sub-PCs will partition from this PC. For example, as indicated in Fig. 3, the PCs for the node n_7 are the 3 domains labeled as PC_7^1, PC_7^2, and PC_7^3 and marked with different colors for easy identification, while the PCs for the node n_{14} are the domains labeled as PC_{14}^1, PC_{14}^2. It can be concluded that for each PC cut by n cracks, no more than 2^n sub-PCs are formed.

In NMM, physical cover function $u_i^l(\tilde{\mathbf{x}})$ is attached on each physical cover. For example, the physical cover function for physical cover PC_7^1 is $u_7^l(\tilde{\mathbf{x}})$, and the physical cover function for physical cover PC_{14}^1 is $u_{14_1}^l(\tilde{\mathbf{x}})$, and for physical cover PC_{14}^2 is $u_{14_2}^l(\tilde{\mathbf{x}})$. The physical

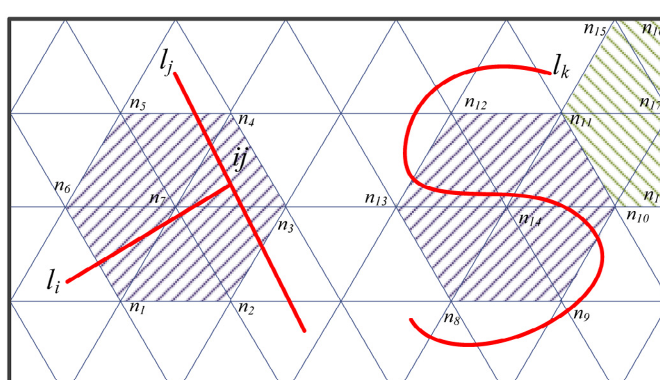

Fig. 2. Mathematical cover system adopted by NMM for the analysis of solid body Ω.