International Journal of Adhesion & Adhesives 29 (2009) 331–341

Contents lists available at ScienceDirect

International Journal of Adhesion & Adhesives

journal homepage: www.elsevier.com/locate/ijadhadh

Analytical models of adhesively bonded joints—Part II: Comparative study

Lucas F.M. da Silva [a,*], Paulo J.C. das Neves [a], R.D. Adams [b], A. Wang [c], J.K. Spelt [c]

[a] Departamento de Engenharia Mecânica e Gestão Industrial, Faculdade de Engenharia da Universidade do Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal
[b] Department of Mechanical Engineering, University of Bristol, Queen's Building, University Walk, Bristol BS8 1TR, UK
[c] Department of Mechanical and Industrial Engineering, University of Toronto, Toronto, Ontario, Canada M5S 3G8

ARTICLE INFO

Article history:
Accepted 11 June 2008
Available online 3 August 2008

Keywords:
Metals
Composites
Lap shear
Stress analysis
Failure load prediction

ABSTRACT

The literature survey presented in Part I describes the major analytical models for adhesively bonded joints, especially for single lap joints. By consulting the summary table given in Part I, the designer can choose from a wide range of models which is the best for a particular situation. However, the information given in the summary table is not sufficient for a proper selection. The designer also needs to know the time required for setting up an analysis and solving it. Another important factor is the accuracy of strength prediction. Therefore, models of increasing complexity were selected from the summary table and a comparative study was made in terms of time requirements and failure prediction for various cases. Three main situations were considered: elastic adherends and adhesive, elastic adherends with nonlinear adhesive, and nonlinear analyses for both adherends and adhesive. The adherends were both isotropic (metals) and anisotropic (composites). The effects of the overlap length and the adhesive thickness were also considered.

1. Introduction

Table 1 of our earlier paper [1] presents a summary of both linear and nonlinear two-dimensional analytical analyses available in the literature. The adhesive joint designer can select from this list the analysis most adequate for a particular situation. However, the true test for a model is to predict joint strength for a variety of conditions. The parameters that were considered are the type of adhesive (brittle or ductile), the type of adherend (isotropic with and without yielding, and composites), the overlap length, and the adhesive thickness. Experimental results obtained previously by the authors were compared with models of increasing complexity so as to ascertain the accuracy of joint strength prediction. Only the most representative models of elastic, nonlinear adhesive, and fully nonlinear were selected to simplify the study since most of the analyses require a numerical solution that is difficult to implement. The models chosen were implemented in the computer code MAPLE to allow for a comparison of time required to prepare the analyses and to solve them.

2. Models studied

Most of the models presented in Table 1 of Part I [1] need to be solved numerically. The implementation of these models is often very laborious and so only some of the models were selected. There are three main classes: elastic, nonlinear adhesive, and nonlinear adhesive and adherend. The models selected to cover these three classes were Volkersen [2], Goland and Reissner [3], Bigwood and Crocombe [4], and Frostig et al. [5] for elastic, Hart-Smith [6], Bigwood and Crocombe [7], and Adams and Mallick [8] for nonlinear adhesive, and Wang et al. [9] and Adams et al. [10] for full nonlinear. The model of Adams and Mallick [8] was also used for studying a case with composite adherends.

2.1. Volkersen

The adhesive shear stress distribution (τ) is given by

$$\tau = \frac{P}{bl}\frac{w}{2}\frac{\cosh(wX)}{\sinh(w/2)} + \left(\frac{\psi - 1}{\psi + 1}\right)\frac{w}{2}\frac{\sinh(wX)}{\cosh(w/2)} \quad (1)$$

where P is the applied load, b the joint width, and l the overlap,

$$w^2 = (1+\psi)\phi$$
$$\psi = t_t/t_b$$
$$\phi = \frac{G_a l^2}{E t_t t_a}$$
$$X = \frac{x}{l}, \quad -\frac{1}{2} \leqslant X \leqslant \frac{1}{2}$$

where t_t the top adherend thickness, t_b the bottom adherend thickness, E the adherend modulus, G_a the adhesive shear modulus, and t_a the adhesive thickness. The origin of the longitudinal co-ordinate x is the middle of the overlap.

* Corresponding author. Tel.: +351 225081706; fax: +351 225081445.
E-mail address: lucas@fe.up.pt (L.F.M. da Silva).

0143-7496/$ - see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijadhadh.2008.06.007

Nomenclature

b	joint width
c	half of the overlap length
D	adherend bending stiffness
E	Young's modulus
G	shear modulus
k	bending moment factor
k'	transverse force factor
l	overlap length
M	bending moment
$\bar{P}$	applied tensile load per unit width
P	applied tensile load
x	longitudinal co-ordinate
t	adherend thickness
t_t	top adherend thickness
t_b	bottom adherend thickness
t_a	adhesive thickness
u	horizontal displacement
w	vertical displacement
y	transverse co-ordinate (thickness direction)
γ_e	elastic adhesive shear strain
γ_p	plastic adhesive shear strain
τ_p	plastic adhesive shear stress
v	poisson's ratio
τ	shear stress
σ	tensile stress

2.2. Goland and Reissner

The expression for the adhesive shear stress is

$$\tau = -\frac{1}{8}\frac{\bar{P}}{c}\left\{\frac{\beta c}{t}(1+3k)\frac{\cosh((\beta c/t)(x/c))}{\sinh(\beta c/t)}+3(1-k)\right\} \quad (2)$$

where $\bar{P}$ is the applied tensile load per unit width, c half of the overlap length, t the adherend thickness,

$$\beta^2 = 8\frac{G_a}{E}\frac{t}{t_a}$$

$$k = \frac{\cosh(u_2 c)}{\cosh(u_2 c)+2\sqrt{2}\sinh(u_2 c)}$$

$$u_2 = \sqrt{\frac{3(1-v^2)}{2}}\frac{1}{t}\sqrt{\frac{\bar{P}}{tE}}$$

The expression for the adhesive peel stress is

$$\sigma = \frac{1}{\Delta}\frac{\bar{P}t}{c^2}\left[\left(R_2\lambda^2\frac{k}{2}+\lambda k'\cosh(\lambda)\cos(\lambda)\right)\cosh\left(\frac{\lambda x}{c}\right)\cos\left(\frac{\lambda x}{c}\right) + \left(R_1\lambda^2\frac{k}{2}+\lambda k'\sinh(\lambda)\sin(\lambda)\right)\sinh\left(\frac{\lambda x}{c}\right)\sin\left(\frac{\lambda x}{c}\right)\right] \quad (3)$$

where

$$\lambda = \gamma\frac{c}{t}$$

$$\gamma^4 = 6\frac{E_a}{E}\frac{t}{t_a}$$

where E_a is the adhesive Young's modulus,

$$k' = \frac{kc}{t}\sqrt{3(1-v^2)\frac{\bar{P}}{tE}}$$

$$R_1 = \cosh(\lambda)\sin(\lambda)+\sinh(\lambda)\cos(\lambda)$$

$$R_2 = \cosh(\lambda)\sin(\lambda)+\sinh(\lambda)\cos(\lambda)$$

$$\Delta = \tfrac{1}{2}(\sin(2\lambda)+\sinh(2\lambda))$$

The origin of the longitudinal co-ordinate x is the middle of the overlap.

2.3. Frostig et al.

The SLJ is divided into three regions (see Fig. 1), the left-end side and the right-end side outer adherends (regions 1 and 3) and the overlap (region 2). The governing equations and boundary conditions, and the continuity requirements are derived for each region using the principle of virtual displacements:

$$\partial U + \partial V \equiv \partial W = 0 \quad (4)$$

where ∂U and ∂V are the internal and external virtual work, and ∂W is the total virtual work. One of the main advantages of Frostig et al.'s model is the ability to partition easily the joint for the inclusion of more than one adhesive or variations in the adherend geometry. The general expression that gives the natural boundary conditions at a point between regions i and i+1 (i = 1, 2) is [5]

Table 1
Comparison of the analytical models in terms of time and computer power requirements

Model	Time (using MAPLE)		Minimum computer power needed
	Preparation (min)	Analysis (s)	
Volkersen [2]	30	0.5	Scientific calculator
Goland and Reissner [3]	60	1	Scientific calculator
Frostig et al. [5]	300	15	Computer
Hart-Smith (linear) [6]	90	1	Scientific calculator
Hart-Smith (adhesive nonlinear) [6]	120	2	Computer
Bigwood and Crocombe (linear) [4]	90	0.5	Scientific calculator
Bigwood and Crocombe (adhesive nonlinear) [7]	180	300	Computer
Adams and Mallick (linear)[a] [8]	180	1	Computer
Adams and Mallick (adhesive nonlinear)[a] [8]	300	300	Computer
Adams and Mallick ('effective modulus')[a] [8,15]	180	1	Computer
Wang et al. (fully nonlinear)[a] [9]	300	1800	Computer
Adams et al. [10]	5	0.5	Simple calculator

[a] Not implemented with MAPLE but with a FORTRAN program.