International Journal of Impact Engineering 69 (2014) 69–79

Contents lists available at ScienceDirect

International Journal of Impact Engineering

journal homepage: www.elsevier.com/locate/ijimpeng

Scaling dynamic failure: A numerical study

T. Noam, M. Dolinski, D. Rittel*

Faculty of Mechanical Engineering, Technion, 32000 Haifa, Israel

ARTICLE INFO

Article history:
Received 11 October 2013
Received in revised form
22 January 2014
Accepted 16 February 2014
Available online 28 February 2014

Keywords:
Adiabatic shear
Scaling
dynamic failure
Strain energy density
Blast
Penetration

ABSTRACT

Scaling failure in blast loaded structures is considered to be impossible with the known scaling laws when using fracture-mechanics based (fracture toughness) considerations (Jones, 1989). We will show in this research that scaling failure becomes possible when 2 alternative competing criteria are used, namely: *maximum normal stress* to describe separation (cracking) and a *strain energy density-based criterion* that describes adiabatic shear failure. Numerical simulations of two test-cases were carried out: Failure of circular clamped plates under close range, air blast loading, and penetration experiments.

This study shows that both the prototype and small-scale model undergo scaling for those failure criteria. This study presents a new alternative to the scaling of structural failure under dynamic loading conditions, which is both simple and efficient.

1. Introduction

From an engineering perspective, the usage of small-scale models can be very effective, as it saves both time and expensive full-scale experiments. By using a small-scale model, one could greatly decrease the difficulty of setting up an experiment, which aside from financial considerations, might also be hazardous (violent blast for instance).

Scaling structural failure in dynamic problems is considered to be impossible while using the established scaling laws if one assumes use of a fracture-mechanics criteria [1]. In order to successfully scale a prototype (full-scale), one must make sure that both the geometry of the model (small-scale) and the failure mechanism used for the model follow the scaling laws. It is discussed in Section 2 that this is not possible for a fracture-mechanics based (fracture toughness) criterion. Scaling a dynamic failure problem therefore requires an alternative failure mechanism that is not based on fracture-mechanics. Adiabatic shear banding (ASB) is one of the many failure mechanisms that occur in dynamic (high strain-rate) loading situations. The homogeneous deformation tends to localize into a narrow band (ASB) followed by catastrophic failure [2]. The typical time scale involved is short, so that one can consider this failure mechanism as adiabatic, with large associated temperature increases, especially inside the band. Many materials fail by adiabatic shear banding in an uncontrolled and dangerous manner [2,3]. One failure criterion, proposed by Rittel et al. [4], is described by

$$E = \int_0^{\varepsilon_f} \sigma_{ij} \mathrm{d}\varepsilon_{ij}^p \tag{1}$$

where σ and ε are the stress and strain tensors. Such a criterion may remind of the work by Cockcroft and Latham [5], albeit in the specifically dynamic context. However, the major difference is that it is developed within a physical context (driven from microstructural considerations) and specifically applied to the dynamic case.

The strain energy density criterion (critical value of E in Equation (1)) is based on the dynamic deformation energy of cold work, and suggests that the dynamic deformation energy is the governing factor governing ASB formation. In experiments conducted by Rittel et al. [4], it was shown that this energy was constant irrespective of static pre-strain levels before dynamic failure, or interruptions in a dynamic test that minimized thermomechanical coupling effects. According to the criterion, a material point starts to fail when the total strain energy density reaches a critical value, the latter being measurable, and different for each material.

Other known (static or dynamic) failure mechanisms include Gurson's model [6], Grady's fragmentation model [7], and the true ductility criterion by Cockcroft and Latham [5]. Gurson's model is based on the plastic flow of a material that contains voids. This model is complex, and requires a number of constants and

* Corresponding author.
E-mail address: merittel@technion.ac.il (D. Rittel).

http://dx.doi.org/10.1016/j.ijimpeng.2014.02.011
0734-743X/

conditions that one must determine to use this model. An example of a damage criterion, based on a modified version of Gurson's model, can be found in the work of Longère et al. [8]. In the author's words [7], "Grady considered improvements to two aspects of the Mott model of fragmentation [9]: the instantaneous appearance of the fracture and the inability to pin down the statistical properties of the failure strain in the material". Mott's model considers statistical aspects such as standard deviation in fracture strains, that can't necessarily be measured in independent experiments. Grady's model involves a large number of parameters that must be known in order to estimate the failure strain. Although Grady's model has been validated [10], we have not found (widespread) engineering applications of this failure model. The Cockcroft and Latham true ductility criterion [5] for a material is the strain at fracture in an idealized test, in which the stress is always of uniaxial torsion. This criterion does not directly correlate to an energy based criterion, except for ideally plastic materials. Moreover, this criterion is proposed for the static failure case, although sometimes employed in dynamic situations [11]. All of these failure criteria may undergo scaling under certain conditions, but we have not found any mention of scaled applications, or considerations based on these criteria.

In order to fully understand the blast mechanism, we first need to understand the nature of a blast wave. A blast wave is basically a pressure wave, resulting from the release of a large amount of energy, in a small and very localized volume. In our case we discuss only a spherical air burst for simplicity.

The pressure at each and every point is a function of the distance from the explosion point and the strength of the explosion. When checking the effect of a blast on a plate or structure of any sort, it is assumed that only the positive phase be taken into account, as the negative phase can be neglected compared to it. The pressure applied to the plate is [12]:

$$\begin{cases} P_{\mathrm{eff}}(t) = P_{\mathrm{inc}}(t)\cdot\left[1+\cos^2\theta-2\cos\theta\right]+P_{\mathrm{ref}}(t)\cdot\cos^2\theta, & \cos\theta\geq 0 \\ P_{\mathrm{eff}}(t) = P_{\mathrm{inc}}(t), & \cos\theta<0 \end{cases} \tag{2}$$

where the angle of incidence θ is formed by the line from the charge center to the point of interest on the structure and the normal vector of the structure's surface at that point, P_{eff} is the effective pressure applied to the plate, P_{inc} is the incident pressure, and P_{ref} is the reflected pressure.

2. Scaling theory

When using scaling theory to compare between a full-scale *prototype* and small-scale *model*, there are relationships between the parameters of the prototype and the small-scale model that must be fulfilled, as discussed e.g. in Jones [1].

This author mentions two important points of the scaling theory. The first concerns geometrically similar scaling, as applied only to linear elastic solids. The second consists of the Buckingham Π-theorem, which also applies to dynamic cases with inelastic response, being, as such, relevant to this work. The scaling laws remain the same for both subjects, thus they can be used for both elastic and inelastic responses. We will present only the relevant parameters for our problem, where a lower case letter represents the small-scale model and an upper case letter represents the full-scale prototype.

The mass density (ρ), Young's elasticity modulus (E), and Poisson's ratio (ν) must be identical. The geometrical scaling factor β is defined as the ratio between a typical length dimension of the small-scale model and the full-scale prototype:

$$\beta = \frac{l}{L} \tag{3}$$

Consequently, any length dimension in the small-scale model is multiplied by the scaling factor. The other parameters are:

- Geometrical dimensions are proportional to the scaling factor: $l = \beta\cdot L$.
- Strains remains the same: $\varepsilon = E$.
- Stresses remains the same: $\sigma = \Sigma$.
- External pressures remains the same, and act at scaled locations: $p = P$.
- Disturbances propagate in the material with the same speed: $c = C$.
- Time is proportional to the scaling factor: $t = \beta\cdot T$.
- Velocities in the structures remain the same: $v = V$

There are a number of phenomena that do not follow the scaling laws, and thus cannot be used when comparing scaled-down cases. The first phenomenon is gravitational force, which cannot be scaled experimentally according to the scaling laws. However, when dealing with a dynamic case that involves high accelerations (for example blast loading), the gravitational forces can often be neglected. Another phenomenon that cannot be scaled is material strain-rate sensitivity. Strain-rate in a small-scale model is larger than in the prototype by $1/\beta$, and thus the stresses in the full-scale prototype and the small-scale model are no longer identical. In order to overcome this, one must choose a material that is not strain-rate sensitive at least for a first extent. The last phenomenon that cannot be scaled is fracture toughness. Fracture toughness is dependent on both stress and crack length, and has units of the form pressure$\cdot\sqrt{\text{length}}$. Thus the fracture toughness of a model must be equal to $\beta^{1/2}$ times that in a prototype and that is normally physically impossible. Consequently, a failure criterion in a scaling problem cannot be fracture-mechanics based. An alternative failure criterion must be used, and this is the key issue addressed in this paper.

Remembering the external pressures must be similar, we will first examine a blast scaling problem. The most common scaling method is known as the Hopkinson (or cube root) scaling law, as found e.g. in Baker [13]. This law determines that two blast waves that are similar, are produced at the same scaled distance if the charges are the same explosive, similar in geometry, but scaled in geometric size (thus weight). For air explosions the Hopkinson scaled parameters are:

$$Z = \frac{R}{E^{1/3}}, \quad \tau^* = \frac{\tau}{E^{1/3}}, \quad \zeta = \frac{I}{E^{1/3}} \tag{4}$$

where Z is the scaled distance, τ^* is the characteristic scaled time of the blast wave, ζ is the scaled impulse, R is the distance from the center of the blast source and E is the source blast energy (E may also be replaced by W which is the weight of the explosive).

This scaling law determines that the peak pressure and velocity are similar in regard to scaling. This means that for the same value of the scaled distance Z, these quantities will remain constant. Remembering the geometrical scaling laws, where geometrical dimensions are scaled by the scaling factor β, one finds that the weight of the explosive material must also be scaled in order for the scaled distance Z to be preserved. This means that the weight of the explosive for the model is the prototype weight multiplied by β^3.

To summarize the main points of the scaling theory of relevance to this work:

- Linear dimensions and time are multiplied by β in a small-scale model.