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Determination of the critical plane by a weight-function method based on the maximum shear stress plane under multiaxial high-cycle loading

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ABSTRACT

A weight function method for the determination of the critical plane is here proposed for the case of specimens under combined bending and torsion in the high cycle fatigue regime. The critical plane is assumed to be coincident with the mean maximum absolute shear stress plane, which is calculated by averaging the instantaneous angle between the specimen axis and the normal to the maximum absolute shear stress plane. Two kinds of weight functions are proposed to determine such a plane. The proposed method to determine the critical plane is verified by employing fatigue data available in the literature in terms of experimental fracture planes, and the multiaxial fatigue life is also predicted by a reformulation of the criterion proposed by Carpinteri et al. to verify the determined critical plane. The results show that the proposed method can be applied to determine the critical plane under both constant and variable amplitude loading.

1. Introduction

Most structural components in service experience complex multiaxial high cyclic loadings so that the directions of both the three principal stresses and the maximum shear stress are time-varying [1]. Development of the efficient and reasonable methods for high cycle fatigue problems is very important, therefore, multiaxial fatigue criteria have been proposed by researchers based on different fatigue behaviors. In high cycle fatigue regime, the criteria can be classified into three categories: stress invariant based criteria [2–4], critical plane based criteria [5–17] and mesoscopic scale based criteria [18–21]. The multiaxial fatigue criteria based on the critical plane, which has clear physical meaning, was often used to evaluate multiaxial fatigue damage. The critical plane is generally defined as the fatigue crack initiate plane based on Stage I cracks. Nevertheless, for hard or brittle metals, owing to the stage II is predominant over stage I, the critical plane is considered as the dominant failure plane. Some researchers determined the critical plane by some stress parameters or the combination of the stresses [7,8,22,23].

Since the principal stress directions are time-varying under non-proportional loading, and such a change in direction has some influences on the fatigue behavior, Macha and Carpinteri [24–30] proposed the weighted mean principal stress directions by the average instantaneous values of the three Euler angles to determine the mean principal stress directions. Based on the stress parameters, Macha and Carpinteri [24–30] proposed various weight functions. In Ref. [24], Macha proposed eleven weight functions resulting from theoretical considerations. These weight functions consider various factors, such as the positions and values of the maximum principal stress, the maximum shear stress, the slope of the fatigue curve and so on. Carpinteri and Spagnoli [27] employed the following weight function to determine the mean principal stress directions:

$$W(t) = \begin{cases} 0 & \text{if } \sigma_1(t) < cf_{-1} \\ \left(\frac{\sigma_1(t)}{f_{-1}}\right)^{m_\sigma} & \text{if } \sigma_1(t) \geqslant cf_{-1} \end{cases} \quad 0 < c \leqslant 1 \tag{1}$$

where $\sigma_1(t)$ is the instantaneous maximum principal stress at time instant t, f_{-1} is the fatigue limit under fully reversed bending, exponent $m_\sigma = -1/m$ depends on the negative slope m of the S–N curve considered, c is a safety factor which varies from 0 to 1. Carpinteri and Spagnoli [27] suggested the value of the parameter c to be 0.5.

Based on the experimental verification, Carpinteri and Spagnoli [27] showed that the direction of normal vector of the experimental fracture plane coincides with the weighted mean maximum principal stress direction determined by Eq. (1). By considering the fatigue crack nucleation and crack propagation develop in different planes, Carpinteri and Spagnoli [27] proposed that the critical plane, distinct from the fatigue fracture plane, could be determined by the following empirical expression:

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Nomenclature

D	total damage
f_{-1}	fatigue limit under fully reversed bending
$m_\sigma = -1/m$	coefficient depending on the slope m of the S–N curve for fully reversed bending
t	time
t_{-1}	fatigue limit under fully reversed torsion
$W(t)$	weight function
β_c	angle that the normal vector of the weighted mean maximum absolute shear stress plane makes with the specimen axis.
$\beta_{critical}$	angle between the normal to the critical plane and the specimen axis
$\beta_{fracture}$	angle between the normal to the experimental or the calculated fracture plane and the specimen axis
$\gamma(t)$	angle between the normal to the maximum absolute shear stress plane and the specimen axis at time instant t
$\hat{\gamma}$	angle between the normal to the weighted mean maximum absolute shear stress plane and the specimen axis
δ	phase angle between the applied shear and normal stresses under combined bending and torsion loading
Δ	generic material plane of the specimen
Δ'	weighted mean maximum absolute shear stress plane
$\sigma(t)$	applied normal stress
$\sigma_1(t)$	instantaneous maximum principal stress at time instant t
σ_f	ultimate tensile strength of the material
$\sigma_{Gough}(t)$	instantaneous Gough equivalent stress at time instant t
$\sigma_{\max c}$	maximum normal stress on the critical plane
$\sigma_{Mises}(t)$	instantaneous von Mises equivalent stress at time instant t
$\tau(t)$	applied shear stress
τ_{ac}	shear stress amplitude on the critical plane
$\tau_{\max}(t)$	instantaneous maximum shear stress at time instant t
$\tau_{\min}(t)$	instantaneous minimum shear stress at time instant t
$\varphi(t)$	instantaneous angle made by the normal vector of the maximum absolute shear stress plane with the specimen axis at time instant t

Subscripts

a	amplitude (of a given stress component)
m	mean value (of a given stress component)
max	maximum value (of a given stress component)
min	minimum value(of a given stress component)

$$\alpha = 45\frac{3}{2}\left[1 - \left(\frac{t_{-1}}{f_{-1}}\right)^2\right] \tag{2}$$

where α is the angle between the weighted mean maximum principal stress direction and the normal to the critical plane, and t_{-1} is the fatigue limit under fully reversed torsion.

As shown by many experimental results, the directions of the maximum shear stress/strain and the maximum principal stress/strain have a strong influence on the fatigue behavior [25]. The critical plane may be determined by the orientations of the maximum shear stress plane. Since the maximum shear stress planes are time-varying under non-proportional loading, the weighted mean maximum shear stress plane should be used to determine the critical plane.

The objective of this paper is to propose a weight function method, based on the maximum shear stress plane, to determine the critical plane under combined bending and torsion loading. Firstly, the obtained fatigue critical plane orientations by experimental observation are used to verify the predictive capability of the proposed weight function method. Then, the fatigue experimental data in the references are used to validate the capability of the proposed method for predicting fatigue life by a reformulation of the criterion proposed by Carpinteri and Spagnoli [27] under constant and variable amplitude loading conditions.

2. The proposed weight function method

As with many other critical plane approaches, the determination of the critical plane orientation in the present study is based on the Stage I process. Stage I cracks initiate along the maximum shear stress planes, and their propagation is mainly Mode II dominated [31]. The critical plane here is assumed to be coincident with the mean maximum shear stress plane. The maximum shear stress planes are time-varying under non-proportional loading. Hence, in order to determine the mean maximum shear stress plane, a weight function method should be used. Since the positive and negative shear stresses, i.e., the forward and reverse shear deformations, have the same contribution for fatigue damage, the critical plane can be determined by the plane experiencing the weighted mean value of the maximum absolute shear stress.

In this paper, the parameter $\gamma(t)$, which is defined as the angle between the normal to the maximum absolute shear stress plane and the specimen axis at time instant t, is used to determine the mean maximum absolute shear stress plane $\hat{\gamma}$:

$$\begin{cases} \hat{\gamma} = \frac{1}{W}\sum_{t=1}^{N}\gamma(t)W(t) \\ W = \sum_{t=1}^{N}W(t) \end{cases} \tag{3}$$

where $W(t)$ is the weight function which will be explained later in Section 2.2, and $\gamma(t)$ can be obtained by:

$$\gamma(t) = \frac{1}{2}arctg\left[abs\left(\frac{\sigma(t)}{2\tau(t)}\right)\right] \tag{4}$$

where $\sigma(t)$ and $\tau(t)$ are the applied normal and shear stresses, respectively.

2.1. Procedure to determine the critical plane orientation by a weight function method

In this section, the procedure for the determination of the weighted mean maximum absolute shear stress plane by two different weight functions is presented to determine the critical plane under combined bending and torsion with constant and variable amplitude loadings.

As shown by Brown and Miller [32], the Case A and the Case B cracks correspond to cracks propagate on the surface and inwards from the surface, respectively. Since the named Case A regards crack occurs under combined bending and torsion loading, the angle θ equals to $\pm\pi/2$, as shown in Fig. 1. Therefore, the three-dimensional case can be reduced to a two-dimensional problem. Thus, the stresses on the plane Δ, whose unit normal vector $\mathbf{n}$ makes an angle φ with the axis of the specimen, can be expressed as: