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# A new path-dependent fatigue damage model for non-proportional multi-axial loading

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ABSTRACT

This paper presents a new path-dependent multi-axial fatigue damage model which is formulated based on an incremental form of Moment of Load Path (MLP) on either $\sigma - \sqrt{\beta}\tau$ stress plane or $\varepsilon - \sqrt{\beta^{\varepsilon}}\gamma$ strain plane. The resulting MLP-based fatigue damage parameter can be shown to be related to an integral form of strain energy densities contributed by normal and shear deformation and each weighted by a path-dependent function. Then, the MLP-based damage parameter in terms of either equivalent stress range or strain range, in conjunction with path-dependent maximum-range cycle counting procedure (Dong et al., 2010; Wei and Dong, 2010), has been shown effective in correlating a large amount of test data obtained under non-proportional multi-axial loading conditions both for welded joints under stress-controlled conditions in high cycle fatigue regime and non-welded components under strain-controlled conditions in low-cycle regime.

## 1. Introduction

Engineering structures are often subjected to multi-axial cyclic stresses during service [1–3]. One common type of multi-axial stress state occurs where there is a sudden change of geometry such as at notches or welded joints, as a result of geometric constraints. Another source for multi-axial stress state is due to external multi-axial loading conditions leading to a stress state that is multi-axial in nature, such as that in a shaft component under both bending and torsion. The first type of multi-axial stress state is largely proportional, in which stress components vary proportionally with each other over time and the corresponding principal stress directions remain unchanged. Since the peaks and valleys of each stress component history occur at the same time for proportional loading, an effective stress range formulated using component stress ranges, e.g., in the form of von Mises stress range have been shown to be effective in fatigue damage modeling, as shown by [1,4,5]. Furthermore, conventional cycle counting methods such as Rainflow cycle counting can still be used by tracking a given time history of one of the stress components and scaling the rest. If stress components at a given material point vary independently or with a clearly defined phase shift angle in sinusoidal variation over time, non-proportionality effects on fatigue damage must be considered, as pointed out by numerous researchers, such as by Sonsino and Kueppers [6] and Yousefi et al. [7] for welded joints and Itoh et al. [8] on plain tube specimens, among others.

It has been observed that non-proportional loading induced fatigue damage depends upon both load path and material [5–15]. Various experimental studies [6–10] have shown that fatigue damage as a result of load-path non-proportionality can be more significant in some materials and to a less degree or even showing a reduced damage in others [13–15]. To deal with the complexity of non-proportional multi-axial fatigue, two key questions must be addressed: (a) how to formulate an effective fatigue damage parameter that is capable of capturing both load path and material effects; (b) how to perform cycle counting against independent component stress histories. To a large extent, both questions are inter-related and must be addressed concurrently when dealing with general variable amplitude multi-axial stress histories.

### 1.1. Fatigue damage parameter

Within the confine of constant amplitude multi-axial loading conditions, such as sinusoidal stress histories of shear and normal stress components with a clearly defined phase shift between them, or some simple path patterns that are repeated during fatigue testing, various fatigue damage parameter definitions have been investigated by numerous researchers [6,8,9,16–18] in the literature. Among them, Dong and Hong [18] proposed a Modified

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## Nomenclature

| | |
|---|---|
| $E$ | Young's modulus |
| $G$ | shear modulus |
| $\upsilon$ | Poisson's ratio |
| $\sigma$ | axial stress |
| $\tau$ | shear stress |
| $\sigma_n$ | nominal axial stress |
| $\tau_n$ | nominal shear stress |
| $\sigma_s$ | bending structural stress |
| $\tau_s$ | shear structural stress |
| $\varepsilon$ | axial strain |
| $\gamma$ | shear strain |
| $\beta$ | fatigue life equivalency factor between tensile stress and torsional stress ($\beta = 3$ taken from literature) |
| $\beta^\varepsilon$ | fatigue life equivalency factor between tensile strain and torsional strain ($\beta^\varepsilon = 1/3$ taken from literature) |
| $\overline{AB}$ | proportional load path |
| $\widetilde{AB}$ | non-proportional load path |
| $\overset{\frown}{AB}$ | semi-circular load path |
| $D$ | total multi-axial fatigue damage of one half cycle |
| $D_P$ | fatigue damage caused by reference proportional load path |
| $D_{NP}$ | fatigue damage caused by non-proportionality of load path |
| $D_{Max}$ | maximum possible non-proportional damage within one half cycle |
| $dD_{NP}$ | non-proportional fatigue damage caused by load path increment, $ds$ |
| $g_{NP}$ | non-proportionality damage factor defined on $\sigma - \sqrt{\beta}\tau$ stress plane |
| $g^\varepsilon_{NP}$ | non-proportionality damage factor defined on $\varepsilon - \sqrt{\beta^\varepsilon}\gamma$ strain plane |
| $\alpha$ | material sensitivity parameter to load-path non-proportionality defined on $\sigma - \sqrt{\beta}\tau$ stress plane |
| $\alpha^\varepsilon$ | material sensitivity parameter load-path non-proportionality defined on $\varepsilon - \sqrt{\beta^\varepsilon}\gamma$ strain plane |
| $\Delta\sigma_e$ | effective stress range along a proportional path or the distance between two extreme positions within one half cycle defined on $\sigma - \sqrt{\beta}\tau$ stress plane |
| $\Delta\varepsilon_e$ | effective strain range along a proportional path or the distance between two extreme positions within one half cycle defined on $\varepsilon - \sqrt{\beta^\varepsilon}\gamma$ strain plane |
| $\Delta\sigma_{NP}$ | MLP-based equivalent stress range |
| $\Delta\varepsilon_{NP}$ | MLP-based equivalent strain range |
| $p(\sigma,\tau)$ | weight function against tensile strain energy density increment ($\sigma d\varepsilon$) |
| $q(\sigma,\tau)$ | weight function against shear strain energy density increment ($\tau d\gamma$) |
| $\eta$ | length ratio of minor axis length over major axis of an elliptical load path |
| $f_b$ | loading frequency of bending stresses |
| $f_t$ | loading frequency of torsional stresses |

Gough Ellipse model in which a fatigue damage parameter is analytically formulated as a function of phase angle ($\delta$) shift between normal and shear stress histories, if both stress histories can be expressed as synchronous sinusoidal wave forms. A good correlation was achieved by examining both proportional and non-proportional test data [6,7,19] obtained from welded joints. However, the Modified Gough Ellipse model is only applicable to constant amplitude non-proportional loading conditions with a clearly defined phase angle. Sonsino and Kueppers [6] showed that non-proportionality induced fatigue damage can be captured by an integral formulation of shear stress over all planes, referred as an Effective Equivalent Stress Hypothesis (EESH). Again, the proposed parameter in [6] can only be applied to constant amplitude sinusoidal loading with a known phase angle between two stress components. Itoh et al. [8] proposed an equivalent non-proportional strain range definition based on principal strain range. A non-proportionality related parameter that takes into account of the rotation of the principal strain axis was used to formulate their fatigue damage parameter. Although a reasonable correlation of non-proportional low cycle fatigue data was demonstrated by Itoh et al. [8], it should be pointed out that their cycle definition for some of the load cases seems questionable, e.g., for similar type of "cross" load patterns, some cases being considered as two cycles while others as one cycle [8]. In addition, their material sensitivity parameter may only be applicable for low cycle fatigue applications.

Without directly addressing the need for a consistent cycle-counting procedure, one important category of non-proportional fatigue damage models is of critical plane type. Among various proposed critical plane models, Findely's stress-based model [20], Brown-Miller's strain-based model [21] and Fatemi–Socie's strain–stress-based model [17] are perhaps the most widely investigated ones. However, a common and non-trivial issue associated with these critical plane models is how to determine shear stress range, $\Delta\tau$ or shear strain range, $\Delta\gamma$ on a potential critical plane since both shear stress and shear strain change their magnitudes and directions along non-proportional load path. As a case in point, a closed irregular load Path $\widetilde{AB}$ (thick lines) shown in Fig. 1 represents non-proportional load path of shear stresses on one potential critical plane. It is not straightforward and an easy task to determine $\Delta\tau$ for load path $\widetilde{AB}$ in Fig. 1. Various approximate methods [22] such as Minimum Circumscribed Circle (MCC), Longest Chord (LC) and Longest Projection (LP) have been proposed. However, none of these methods could truly differentiate between a proportional path i.e., $\overline{AB}$ and the actual non-proportional load path $\widetilde{AB}$ as illustrated in Fig. 1. For instance, consider non-proportional path $\widetilde{AB}$ in Fig. 1, MCC method involves finding a minimum circle that circumscribes the load path, which yields a radius ($R$) as the shear stress amplitude. However, as can be seen in Fig. 1, the use of the radius ($R$) for representing fatigue damage caused by the non-proportional load path $\widetilde{AB}$ ignores all non-proportional path excursions away from the straight line load path $\overline{AB}$. Li et al. [23] proposed a Minimum Circumscribed Ellipse (MCE), also shown in Fig. 1 for shear stress range calculation. Then, an equivalent shear stress amplitude is defined as the root mean square of the semi-major axis (denoted as $a$ in Fig. 1) and semi-minor axis (denoted as $b$ in Fig. 1). However, as shown by Skibicki [3], MCE has been demonstrated to be inadequate in differentiating various non-proportional load paths that share the same MCE. Therefore, an effective path-dependent fatigue damage parameter within the context of critical plane methods remains to be fully resolved.

### *1.2. Fatigue cycle definition*

Another fundamental question in the treatment of multi-axial fatigue damage in non-proportional variable amplitude loading is how to define a fatigue cycle. In the context of critical plane approach, Bannantine and Socie [24] proposed that component