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International Journal of Adhesion & Adhesives 28 (2008) 237–255

International Journal of Adhesion & Adhesives

www.elsevier.com/locate/ijadhadh

On using a penalty-based cohesive-zone finite element approach, Part I: Elastic solution benchmarks

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Received 2 November 2006; received in revised form 1 March 2007; accepted 18 June 2007
Available online 12 September 2007

Abstract

This paper develops and demonstrates a novel penalty methodology for enhancing the use of the cohesive-zone method (CZM) in finite element models to analyze crack initiation and propagation of surface-bonded structures. For many industrial uses, the CZM finite element approach is troublesome because it is a 3-parameter model depending on critical energy release rate, critical limiting maximum stress and the shape of the traction-separation law. The penalty framework described in the current work maps the CZM approach to fit within the classic Griffith energy release method which is dependent solely on the single material parameter of critical energy release rate. This penalty approach is demonstrated for two generalized problems: double cantilever beam (DCB) analysis and single-arm peeling of very thin elastic substrates. Comparisons with several analytical and pseudo-analytical benchmarks demonstrates how to utilize this new technique as well as the accuracy of the resulting finite element analysis (FEA) solutions for these nonlinear crack propagation and peeling problems.

Keywords: Finite element stress analysis; Peel; Fracture mechanics; Cohesive elements

1. Introduction

Many modern structures rely on surface-bonding to hold components together. In some cases the bonds are intended to be permanent, but in others they are not. Modern automobiles have a variety of surface-bonded components that could fail in a severe crash, jeopardizing structural integrity of the vehicle. If the components are inside the passenger cabin, they could become dangerous projectiles. On a very different scale of complexity are packaging problems such as designing a robust pretzel bag that has sufficient seal-force integrity to survive shipping but will physically open readily at a load that can accommodate a broad range of people. Similar challenges are faced when designing a peelable lid on a semi-rigid food container that must survive internal pressure and shipping loads, yet peel open cleanly without partially delaminating or tearing. These very different classes of products have one thing in common—the need to understand and predict the onset and *propagation* of failure in a bonded joint.

Classical finite element analysis (FEA) methods for fracture mechanics [6], linear elastic fracture mechanics (LEFM) and elastic/plastic fracture mechanics (EPFM), have generally had very limited success at analyzing such complex problems. However, a newer approach based on the *cohesive-zone method* (CZM) enables generalized finite element modeling of crack propagation problems, including the analysis of both crack growth onset and its ongoing propagation through a structure along a defined surface [1,4,7,8,10,11]. While the technology is termed "cohesive zone", it can actually be applied to both adhesive and cohesive fractures.

CZM technology is fundamentally based on energy principles and a traction-separation law between two surfaces. For an isotropic representation, the law can be represented by three parameters: critical energy release rate (G_c), critical limiting maximum stress, and the shape of the traction-separation law. In a typical application, the modeling approach relies on a single layer of *cohesive elements* to represent a "bond", where the term bond is

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doi:10.1016/j.ijadhadh.2007.06.003

generically defined to potentially represent an adhesive joint, fracture surface interface, or similar construct. The bond can be thick or it can be of zero thickness. The cohesive elements are connected, top and bottom, to the adjoining bodies by either sharing common nodes or through an interface constraint. During the analysis, the cohesive elements carry loads to connect the two parts together until such a point in the solution exists for which conditions mandate the initiation of damage and potentially complete failure within any given cohesive element(s). These criterions are assessed on an element by element basis continually throughout the solution. The actual modeling techniques and syntax utilized in an FE analysis with cohesive elements is slightly dependent on the actual FE code that is utilized, although most of the principals are universally applicable. For the calculations in this paper, the FE code utilized is ABAQUS Version 6.5. In ABAQUS, a broad set of cohesive elements features and options are provided so that the user has the ability to specify criterion for each phase of deformation within the cohesive element (undamaged elastic response, damage initiation, and failure).

For many industrial uses, attempting to perform a "typical" CZM FEA is troublesome because of the difficulty in obtaining all the traction-separation law parameters. As an example, consider the peeling of two heat-sealed polymeric films. The common material data that will be available is the apparent uniaxial membrane stress/strain tensile curve of each film, possibly some knowledge of the individual film layer constituents (materials and thickness), and the total force required to peel apart the two films at some peel angle. With this limited information, one can utilize analytical methods [14] to obtain an estimate of the critical energy release rate, G_c. Determination of the critical limiting maximum stress and the shape of the traction-separation law (required for a "full CZM" analysis) are much more difficult and often deemed impractical. This scenario, where only the critical energy release rate is known, is very common in an industrial setting. This paper develops and demonstrates a novel penalty methodology that enables the use of CZM within a FEA where the "bond" is characterized only by the critical energy release rate, G_c.

2. Generalized Griffith energy criterion

This section provides a brief overview of the generalized Griffith energy criterion used to characterize crack propagation. This will provide a foundation for developing benchmarks and for understanding the underpinnings of the "penalty-CZM" approach as applied in a FE analysis.

Fig. 1(a) depicts two beams bonded together. The bonding method could have utilized an adhesive such as "glue", ultrasonic welding, conventional welding, thermal bonding via heat sealing, or other technologies. As the tips of the beams are pulled apart, a point in the deformation history arises after which a crack extends through some portion of the bonded area. Performing an energy balance of the system as the crack propagates between states 1 and 2 in Fig. 1(a) requires

$$\Delta U_{\text{ext}} = \Delta U_{\text{int}} + \Delta U_{\text{c}}, \tag{1}$$

where ΔU_{ext} represents the energy change from the externally applied load P, ΔU_{int} denotes change in stored energy in the two double cantilever beam (DCB) arms, and ΔU_{c} represents the energy released as the crack extends a distance Δa. Normalizing Eq. (1) by the beam width b and crack growth Δa, and then taking the limit as $\Delta a \to 0$, we obtain

$$\frac{1}{b}\left(\frac{\mathrm{d}U_{\text{ext}}}{\mathrm{d}a}\right) = \frac{1}{b}\left(\frac{\mathrm{d}U_{\text{int}}}{\mathrm{d}a}\right) + G_{\text{c}}, \tag{2}$$

where the critical energy release rate, G_c, of the bond is defined as

$$G_{\text{c}} = \frac{1}{b}\left(\frac{\mathrm{d}U_{\text{c}}}{\mathrm{d}a}\right). \tag{3}$$

Re-arranging Eq. (2) yields the classical form of the critical fracture energy as

$$G_{\text{c}} = \frac{1}{b}\left(\frac{\mathrm{d}U_{\text{ext}}}{\mathrm{d}a} - \frac{\mathrm{d}U_{\text{int}}}{\mathrm{d}a}\right). \tag{4}$$

The critical energy release rate is commonly referred to as the critical release energy or critical fracture energy. It is a material parameter that characterizes the amount of energy a bond or fracture surface dissipates per change in unit crack growth per unit depth. These equations are equally applicable to general crack growth within a single material (Fig. 1(b)). It is important to note that in using this energy-based approach to analyze the crack and its propagation, we are implicitly taking a global or smeared approach to the problem, as opposed to a highly local or detailed analysis that is utilized with classical fracture mechanics methods derived around stress intensity factors.

It is useful to relate the critical fracture energy, G_c, to other quantities such as external loads and displacements of the structure. Fig. 1(c) depicts a generic curve that characterizes the externally applied load, P, as a function of opening displacement, y. As depicted, the load and displacement increase until such time when the crack begins to grow. Computing the change in external energy due to crack growth from this generic curve, ignoring second-order terms, and substituting that result into Eq. (4), leads to:

$$G_{\text{c}} = \frac{1}{b}\left(P\frac{\mathrm{d}y}{\mathrm{d}a} - \frac{\mathrm{d}U_{\text{int}}}{\mathrm{d}a}\right). \tag{5}$$

In general, the internal energy term of Eq. (5) can be defined as the sum of elastic and inelastic internal energies. Specific forms relating internal energy to externally applied loads or displacements are problem dependent. Sections 3 and 4 will derive these additional relationships for the two benchmark problems studied.