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Bending moment calculation for single lap joints with composite laminate adherends including bending-extensional coupling

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ABSTRACT

This work deals with the analytical determination of section forces and moments in adhesively bonded single lap joints with composite laminate adherends including bending-extensional coupling. The analysis is also valid for unbalanced joints and the adhesive thickness is taken into account in the model. Several types of boundary conditions can be applied, e.g. simply supported ends, fixed ends, bonded doubler and single strap joints. Various configurations are studied and the results are compared to finite element analyses and other analytical approaches. Good agreement is achieved over a wide range of configurations. A closed form analytical solution is given for the section forces and moments at the overlap ends.

1. Introduction

Lightweight structures are increasingly used in all kinds of engineering fields. With the ongoing development of lightweight components and the widespread use of fiber reinforced composite structures a main concern is the assembling of the components parts. Adhesive bonding has proven advantageous compared to traditional assembly methods and could become a supplementary technique for the connection of component parts in structural mechanics [1]. Due to the typically small wall thicknesses in lightweight structures, classical bonding techniques such as riveting are not applicable without additional effort. In this regard fiber reinforced composite structures are of special interest since they are mostly designed thin-walled and techniques as riveting are often associated with damaging the load bearing fibers [2]. Furthermore adhesively bonded joints have already been successfully used in secondary non-load-bearing structural parts. Repairs on fractured structural components are often bonded adhesively (strap repairs) [3] and adhesive bonding is a common technique in microelectronics where chips are bonded on a substrate (bonded doublers) [4]. To gain further acceptance the analysis of adhesive bonded joints must develop to a level which guarantees a reliable strength prediction and joint design. Up to now this is a matter of ongoing research.

For the analysis of adhesive joints numerical [5] and analytical [16,17] approaches exist. To allow for an efficient analysis as it is necessary in early design stages such as pre-dimensioning or in optimization processes analytical models are often used advantageously.

Most of the current approaches for the stress analysis of adhesive joints model the joint as a three layer structure (adherend/adhesive/adherend) often refered to as sandwich-type model. Numerous works on these types of models exist for many different kinds of joint configurations. The classical works of Volkersen [6], Goland and Reissner [7] and Hart-Smith [8] treat single lap joints. Other authors are engaged in double lap joints [9] or strap repairs [10]. Weißgraeber and Becker [11] propose a failure load prediction for single lap joints with brittle adhesives based on a sandwich-type model. The most general sandwich-type models are proposed by Bigwood and Crocombe [12] and recently by Weißgraeber et al. [13] and Liu et al. [14]. These models treat only the overlap region but are valid for all possible kinds of joint configurations by applying the associated boundary conditions. The requirement of section forces and moments as boundary conditions at the overlap ends is common to all sandwich-type models. In the most general case these are the normal forces, shear forces and bending moments for the top and bottom adherend on the left and right ends of the overlap region, respectively. The importance of these forces and moments as boundary conditions on the overlap ends is emphasized by Luo and Tong [15] who correctly state that the edge moments are one key parameter for analyzing bonded joints. A comprehensive review of adhesive joint models and the computational costs of each is given by da Silva et al. [16,17]. In previous works two different approaches to calculate the section forces and moments at the overlap ends were

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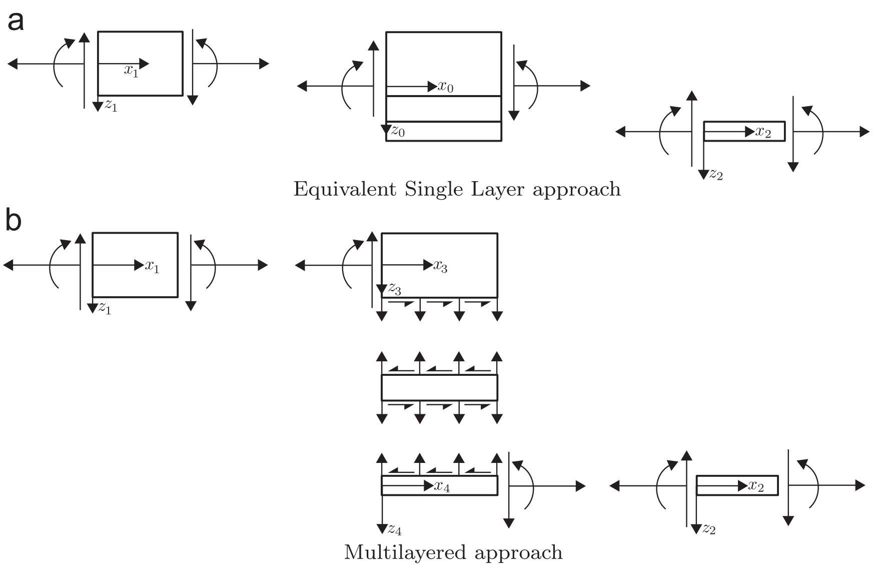

Fig. 1. (a) Equivalent Single Layer approach (ESL): Each of the three sections is modeled as an equivalent single layer beam. This leads to three connected beams with discontinuous midplanes (b) Multilayered approach (MUL): The overlap section is modeled as two beams interconnected by the adhesive layer, leading to continuous midplanes. (a) Equivalent Single Layer approach (b) Multilayered approach.

Table 1
Comparison of different analytical models for prediction of the section moments. Abbreviations: GR: Goland and Reissner (1944); HS: Hart-Smith (1973); CC: Chen and Cheng (1991); OP: Oplinger (1991); LT04: Luo and Tong (2004); GUO: Guo et al. (2006); LL: Li and Lee-Sullivan (2006); BZH: B. Zhao (2009); XZH: X. Zhao (2010); WAH: Wah (1973); REN: Renton (1973); DEL: Delale (1981); YP: Yang and Pang (1983); NEV: das Neves (2009); LT09: Luo and Tong (2009); PRES: Present approach. ESL: Equivalent Single Layer approach; MUL: Multilayered approach; Nonlin.: Nonlinear geometry; l_1, l_2: non-overlaping adherend length; h_1, h_2: adherend thicknesses; A_{11}: extensional stiffness; D_{11}: bending stiffness; B_{11}: bending extensional coupling stiffness; Iso.: Isotropic; Adh.: Adhesive; t: Adhesive thickness; disp. ass.: displacement assumptions; Bern.: Bernoulli assumptions; Timo.: Timoshenko assumptions.

		Approach		Nonlin.	Adherend					Adh.	disp. ass.	
		ESL	MUL	Overlap	$l_1=l_2$	$h_1=h_2$	A_{11}, D_{11}	B_{11}	Iso.		Bern.	Timo.
GR	[7]	x	–	x	x	x	–	–	x	–	x	–
HS	[8]	–	x	–	x	x	x	–	x	x	x	–
CC	[18]	x	–	x	∞	–	–	–	x	–	x	–
OP	[19]	–	x	x	x	x	–	–	x	x	x	–
LT04	[20]	–	x	–	x	x	–	–	x	x	x	–
GUO	[21]	x	–	x	x	x	–	–	x	–	x	–
LL	[22]	–	x	–	x	x	–	–	x	x	x	–
XZH	[23]	x	–	–	–	–	–	–	x	–	x	–
WAH	[24]	–	x	–	x	–	x	–	x	x	x	–
REN	[25]	x	–	x	–	–	x	–	x	–	x	–
NEV	[26]	–	x	–	–	–	x	–	x	x	–	x
YP	[27]	–	x	–	–	–	x	x	x	x	–	x
LT09	[28]	–	x	x	x	x	x	x	x	x	x	–
PRES		x	–	x	–	–	x	x	x	x	–	x

taken, the Equivalent Single Layer approach (ESL) and the Multilayered approach (MUL). For the ESL approach [7,18,21,23,25] (Fig. 1(a)) the whole single lap joint is comprised of three beam sections with midplane-discontinuities at the overlap ends. For the MUL approach [8,19,20,22,24,26–28] (Fig. 1)(b) the overlap region is further partitioned. For this approach the overlap region is composed of two adherend beams interconnected by a weak interface model of the adhesive layer. The section forces and moments at the overlap ends are often not solved explicitly for the MUL approach. But they are used implicitly for the derivation of the adhesive stresses. Due to the modeling assumptions the overlap region in the ESL approach is modeled too stiff which is the reason why the MUL approach is usually more accurate for long overlaps, thick adhesives or thin adherends.

Most of the analytical works treat balanced joints [7,8,19–22,28], i.e. the overlap region is symmetric (geometry and material) about the midplane of the adhesive layer and the adherends have the same length. Further, the adhesive thickness [7,18,21,23,25] and for composite adherends the bending-extensional coupling [7,8,18–26] is often neglected. In Table 1 a comprehensive summary is given to compare and categorize established analytical works. Focusing on the 7th column of Table 1, it becomes evident that bending-extensional coupling in terms of the coupling stiffness B_{11} is neglected by most authors. Besides the present study solely Yang and Pang [27,29] and Luo and Tong [28] deal with this quantity.

Yang and Pang [27,29] formulate a model for single lap joints under tension with simply supported ends. First Order Laminated Plate Theory is used to derive the governing equations for the adherends with bending-extensional coupling included. The influence of the transverse deflection and therewith nonlinearity on the bending moment curvature is only regarded in the overlap-free regions. Furthermore, the solution is obtained with a Fourier series approach for the adhesive normal stresses. With the