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Structural fatigue crack growth on a representative volume element under cyclic strain behavior

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ABSTRACT

By introducing parameters λ and ω into the crack tip field, a unified cyclic stress and strain field was first formulated by using the Hutchinson-Rice-Rosengren (HRR) field and the Rice-Kujawski-Ellyin (RKE) field under plane stress states in the present study. On the basis of the plastic strain energy and the linear damage accumulation, two fatigue crack growth models without any artificial parameters were then proposed from a representative volume element of cyclic strain behavior. The fatigue crack growth model included parameters λ and ω which showed the effect of two singularity fields. In addition, a simplified structural fatigue crack growth model was eventually established in terms of the fatigue life of each point on the crack front and the non-self-similar shape evolution law. Finally, the predictions of models are compared with the experimental data and the agreement is found to be fairly good.

1. Introduction

The fatigue crack growth (FCG) is one of the most important issues during both the design process and subsequent monitoring stage for a critical engineering component. It has been recognized that most structural fatigue failures originate from intrinsic and artificial defects such as surface scratches, corner cracks, and notches at fastener holes [1]. After crack initiation, the structural crack growth becomes a key problem for some engineering components. The propagation stage can represent a significant portion of cyclic failure life, especially when these defects are part-through cracks. Therefore, it is extremely necessary to develop an accurate method to predict the fatigue crack growth and the remaining life under detailed crack configurations in a damage-tolerance design for engineering structures.

A crack initiation occurs followed by the fatigue crack propagation. The unstable crack growth to fracture then happens as the crack reaches a critical size [2]. It is well-known that the fatigue crack growth (FCG) rate is statistically acquired through a pure curve fit of finite experimental data. Based on the linear elastic fracture mechanics (LEFM), various relationships between the fatigue crack growth rate da/dN and the stress intensity factor (SIF) range ΔK have been established in different loading conditions for engineering materials. As suggested by Paris and Erdogan [3], $\log(da/dN)$ is approximately linear correlated with $\log(\Delta K)$ in the intermediate range.

Since the material ahead of the crack tip is modeled as an assemblage of uniaxial material element, a number of fatigue crack growth models [4–9] have been proposed by using the low cycle fatigue (LCF) properties. Note that suitable cyclic failure criteria in the vicinity of the crack tip should be employed, such as the fatigue ductility, the plastic strain energy (PSE) and the weighted local strain, etc. However, previous models contained artificially adjustable parameters that were required being determined from tedious experiments or simulations. Although a few models containing material constants have been established to predict the crack growth behavior [8,9], a tricky problem is that material constants are actually difficult to be obtained from the view point of cost.

The present work is aimed at developing a simplified structural fatigue crack growth (SFCG) model based on the representative volume element (RVE) of cyclic strain behavior. First, a unified cyclic stress & strain field ahead of the crack tip, by introducing parameters λ and ω, is obtained in terms of theoretical solutions of the Hutchinson-Rice-Rosengren (HRR) field and the Rice-Kujawski-Ellyin (RKE) field. Second, material elements near the crack tip are presumed to subject to strain cycle fatigue loads. Thus materials in the vicinity of the crack tip can be seen as RVE under cyclic strain conditions. Third, two FCG models are established by using the fatigue failure criteria of the plastic strain energy (PSE) and the linear damage accumulation (LDA), respectively. These models are verified by open-published experimental data. Finally, by

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considering the cycle fatigue failure life of each point on the structural crack front and the non-self-similar crack shape evaluation law [10], a simplified and novel SFCG model is proposed to evaluate and predict the remaining life of an engineering component in-service. It is found that experimental results coincide well with predictions of the SFCG model.

2. A unified cyclic stress & strain field

The HRR field [11] and the RKE field [12,13] can often be used to describe the stress and strain field in the vicinity of the crack tip under plane stress conditions, in which the crack is subjected to the remote tension load. However, cyclic stress and strain fields ahead of the crack tip are required in the evaluation of fatigue crack growth rates, in which material elements of fatigue crack tip are subjected to the remote cyclic tension loads. In this case, the accurate solution of cyclic stress and strain fields is not available in the classical fracture mechanics theory.

To evaluate the response to unloading, reloading and cycle loading ahead of the fatigue crack tip, Rice's plastic superposition theory [14] is adopted in the present study. The preliminary assumption of Rice's plastic superposition is that the components of plastic strain tensor remain the constant proportional to one another at each point in the plastic region. It should be noted that this superposition method may also be taken as a first approximation in the absence of a better accurate theory in the fatigue process zone analysis.

Based on the above analysis, a cyclic stress and strain field along the crack line ($\theta = 0$) can be obtained from the HRR field as follows:

$$\begin{cases} \Delta\sigma = 2\sigma_{yc}\left(\frac{\Delta K^2}{4\alpha_c I\sigma_{yc}^2 r}\right)^{1/(N_c+1)}\tilde{\sigma}_\theta \\ \Delta\varepsilon = 2\frac{\sigma_{yc}}{E}\left(\frac{\Delta K^2}{4\alpha_c I\sigma_{yc}^2 r}\right)^{1/(N_c+1)}(\tilde{\sigma}_\theta - \nu\tilde{\sigma}_r) \\ +2\frac{\sigma_{yc}}{E}\alpha_c\left(\frac{\Delta K^2}{4\alpha_c I\sigma_{yc}^2 r}\right)^{N_c/(N_c+1)}(\tilde{\sigma}_\theta - 0.5\tilde{\sigma}_r) \end{cases} \tag{1}$$

in which $\Delta\sigma$ and $\Delta\varepsilon$ are the stress and the strain range, respectively, $\sigma_{yc} = E\varepsilon_{yc}$ is the cycle tensile yield stress, ε_{yc} is the cycle tensile yield strain, E is the elastic modulus, n_c is the cycle strain hardening exponent ($N_c = 1/n_c$), ΔK is the SIF range, r is the distance from the crack tip, I is the non-dimensional parameter of cycle strain exponent n_c, $\tilde{\sigma}_\theta$ and $\tilde{\sigma}_r$ are non-dimensional distribution functions, v is Poisson's ratio, α_c is the cycle Ramberg–Osgood parameter [15]. While the cyclic Ramberg–Osgood parameter α_c can be taken as:

$$\alpha_c = \frac{2E}{(2K_{unifiedc})^{N_c}(2\sigma_{yc})^{1-N_c}} \tag{2}$$

in which $K_{unifiedc}$ is the unified cycle strain hardening coefficient. Similarly, considering Rice's superposition theory and the modified form of Rice's solution for anti-plane shear (RKE field), the cyclic stress and strain field along the crack line ($\theta = 0$) is given by Eq. (3).

$$\begin{cases} \Delta\sigma = 2\sigma_{yc}\left[\frac{\Delta K^2}{4(1+n_c)\pi\sigma_{yc}^2 r}\right]^{n_c/(1+n_c)} \\ \Delta\varepsilon = 2\varepsilon_{yc}\left[\frac{\Delta K^2}{4(1+n_c)\pi\sigma_{yc}^2 r}\right]^{n_c/(1+n_c)} + 2\varepsilon_{yc}\left[\frac{\Delta K^2}{4(1+n_c)\pi\sigma_{yc}^2 r}\right]^{1/(1+n_c)} \end{cases} \tag{3}$$

In the present study, the cyclic elastic strain range can be neglected because the cyclic plastic strain range is much larger than the cyclic elastic strain range in the fatigue process zone. Therefore the range of cyclic plastic strain is only adopted in the fatigue crack growth analysis.

It is well-known that the cyclic stress range can be described by the power law function of the cyclic plastic strain range.

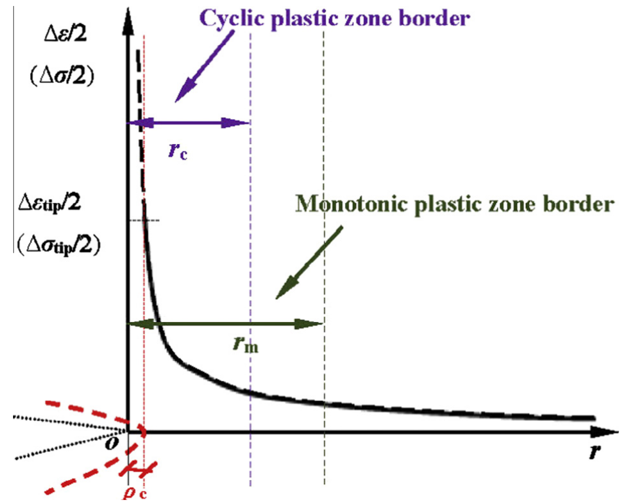

Fig. 1. Distribution of cyclic stress and strain near the crack tip (r_m is the monotonic plastic zone, ρ_c is the crack blunting radius).

Considering Eqs. (1) and (3), a unified cyclic stress and strain field in the vicinity of the crack tip under small-scale yielding can be proposed by introducing parameters λ and ω. In the present study, the effect of two singularity fields on the fatigue crack growth will be investigated by using parameters λ and ω. The elastic strain range is neglected in the unified cyclic stress and strain field. Then, the unified cyclic stress and strain field can be formulated by:

$$\begin{cases} \Delta\sigma = 2K_{unifiedc}\cdot\left(\frac{\Delta\varepsilon_p}{2}\right)^{n_c}\cdot\lambda \\ \Delta\varepsilon_p = 2\frac{\sigma_{yc}}{E}\left(\frac{r_c}{r}\right)^{1/(1+n_c)}\cdot\omega \end{cases} \tag{4}$$

in which

$$\lambda = \begin{cases} \left(\frac{(1+n_c)\pi}{\alpha_c I}\right)^{n_c/(1+n_c)}\tilde{\sigma}_\theta & \text{HRR fields} \\ 1 & \text{RKE fields} \end{cases} \tag{5}$$

$$\omega = \begin{cases} \left(\frac{(1+n_c)\pi}{\alpha_c I}\right)^{1/(1+n_c)}\alpha_c(\tilde{\sigma}_\theta - 0.5\tilde{\sigma}_r) & \text{HRR fields} \\ 1 & \text{RKE fields} \end{cases} \tag{6}$$

$$K_{unifiedc} = \begin{cases} \sigma_{yc}\left(\frac{E}{\sigma_{yc}}\frac{1}{\omega}\right)^{n_c}\cdot\lambda & \text{HRR fields or RKE fields} \\ \frac{\sigma'_f}{\left(\varepsilon'_f\right)^{b/c}} & \text{Low cycle fatigue properties} \end{cases} \tag{7}$$

in which $\Delta\varepsilon_p$ is the plastic strain range, σ'_f and ε'_f are the fatigue strength coefficient and ductility coefficient, respectively, b and c are the fatigue strength exponent and ductility exponent, respectively. The r_c is the cycle plastic zone near the crack tip on the crack front (Fig. 1), which can be described by the following expression:

$$r_c = \frac{\Delta K^2}{8(1+n_c)\pi\sigma_{yc}^2}\left(1 + \frac{3}{2}\sin^2\theta + \cos\theta\right) \text{ plane stress} \tag{8}$$

in which θ is the angular between the crack surface and the radial ahead of the crack tip.

3. FCG models

Eq. (4) exhibits a singularity as $r \to 0$. It is well-known that the singularity is rather unreasonable because the blunting happens ahead of the crack tip under complex material and loading